

# How much to trim? A methodology for calculating Core Inflation, with an application for Brazil

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## RESUMO

O conceito de núcleo da inflação tem ocupado um papel central nas discussões envolvendo políticas macroeconômicas. Apesar disso, nenhuma medida consensual deste conceito existe até agora. Neste artigo, propomos a aplicação da metodologia estatística baseada em médias aparadas, onde o tamanho da apara é baseado em uma referência proporcionada por modelos de componentes estocásticos comuns, os quais acreditamos captar a essência do conceito de núcleo da inflação. Estas medidas são aplicadas ao IPC-FIPE e ao IPCA-IBGE, dois dos principais índices de preços ao consumidor no Brasil, e comentamos os resultados.

**Palavras-chave:** Núcleo da inflação, médias aparadas

## ABSTRACT

The concept of core inflation has been central to the discussions concerning macroeconomic policy management. In spite of this fact, no single measure of this concept has achieved consensus so far. Here, we propose the application of a statistical methodology based on trimmed means, where the amount of trim is based on a benchmark provided by a common stochastic components model, which we believe captures the essence of the core inflation concept. We apply this measure to the IPC-FIPE and IPCA-IBGE, two of the leading consumer price indexes in Brazil, and comment on the results.

**Key words:** core inflation, trimmed means.

**JEL classification:** C22, E31.

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## 1 Introduction

It has been pointed out that there is a discrepancy among the concept of “inflation”, which should represent a “sustained increase in the general price level”,<sup>1</sup> and the regular aggregate price level measures, which are designed to measure the costs of a particular basket of goods and services at a particular point in time. In a recent paper, Delfim Netto provides a concise definition of the relevance of the core inflation concept: “... *it should be relatively stable and able to distinguish among the perturbations produced by transitory effects upon prices (crop failures, energy price shocks, tax raises, public and customs’ tariffs) from the ones resulting from structural pressures related to supply and demand, which can be influenced by monetary policy. The former are transitory, and occur only once. They represent relative price movements which raise (or lower) the general price level, but do not produce an inflationary process. The former are cumulative and their persistence raises some prices initially, create a “expectation” of inflation followed by a spillover to other prices, and finally initiate demands for nominal wages corrections.*”<sup>2</sup> In another recent paper, which provides an excellent survey for the main conceptual issues related to the measurement of core inflation, Wynne states that “*The notion of core inflation has played an important role in the deliberations of monetary policymakers for the past, twenty-five years. However, despite the central role of this concept, there is still no consensus on how best to go about measuring core inflation.*”<sup>3</sup>

The main motivation behind our paper is to propose a methodology for determining the value of the key parameters in one of the well established measures of core inflation.

The justification behind yet another methodology is that it is based on the combination of the idea of trimmed means, which has appealing theoretical justifications, with one concept based on solid statistical concepts, which appears to capture the very nature of the problem at hand: to isolate transient shocks to the prices of particular items entering the general price level computation from factors related to the real fundamentals leading to systematic movements of the general price level. The basic idea of this concept is to extract a common stochastic component from the variations found among the major components of a certain general price index. This is achieved by applying the Kalman Filter in conjunction with maximum-likelihood estimation of a state-space model, in the spirit of Stock and Watson (1991). Although we provide a particular application to two particular consumer price indexes calculated in Brazil, as

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1 Quah and Vahey (1995, p. 1130).

2 Delfim Netto (1999, p. 374), our translation.

3 Wynne (1999, p. 2).

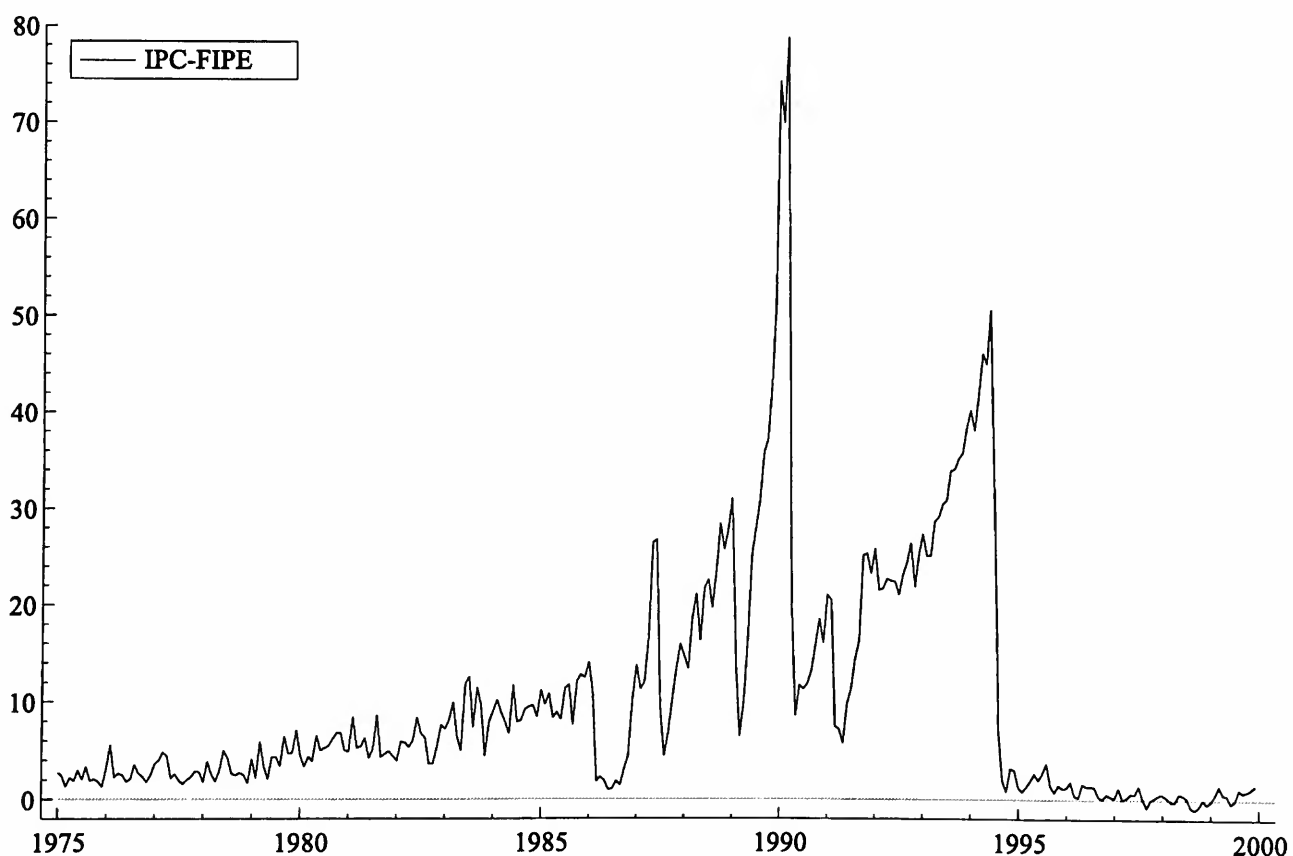
made explicit in the next section, the methodology can in principle be applied to any price index for which there is access to its major components.

## 2 The data

### 2.1 IPC-FIPE

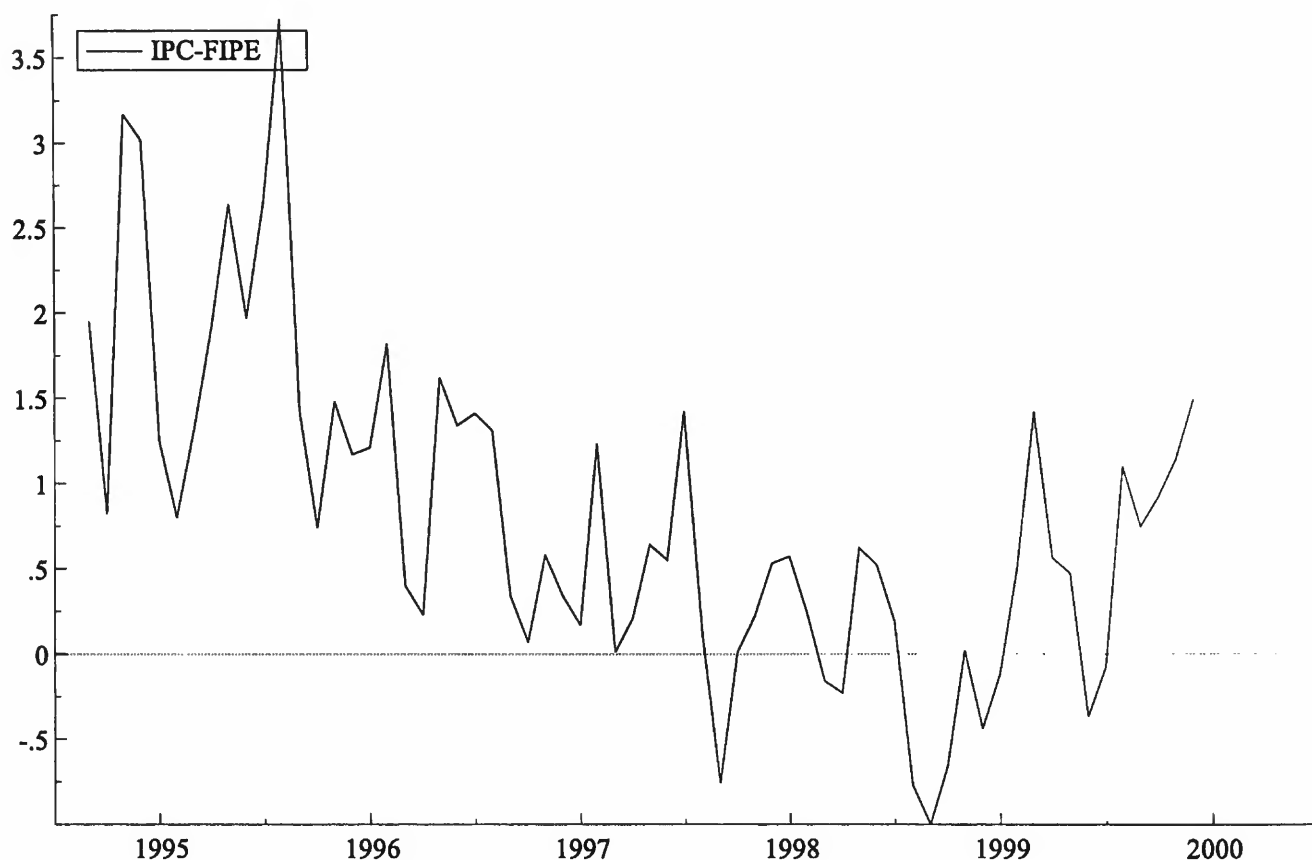
Here we use one of the leading consumer price indices in Brazil, the IPC-FIPE (which stands for índice de preços ao consumidor da Fundação de Instituto de Pesquisas Econômicas). This corresponds to a measure of prices relevant for the modal class of consumers in the City of Sao Paulo. If we take the whole time-series values available, we will face the problem of modelling a series subject to a number of structural breaks and different regimes, corresponding to a series of near hyperinflation periods followed by “heterodox” stabilization plans (which included price freezes and other measures designed to affect prices directly). It is difficult even to visualize the series, because of the large values resulting for the index if we take the whole period. If we look at the rates of variation instead, for the period between January 1975 and December 1999, we can still see the significant structural breaks in the series:

Figure 1



Since our main concern here is to provide a reliable measure of core inflation which can be used for future policy purposes, we restrict our sample to June 1994 to December 1999. This corresponds to the period following the last, and still most successful stabilization plan in the Brazilian economy, the “Real Plan”. Figure 2 below makes it clear the claims of success for this stabilization plan, if measured by reduction of inflation to low levels, especially when compared to the previous periods:

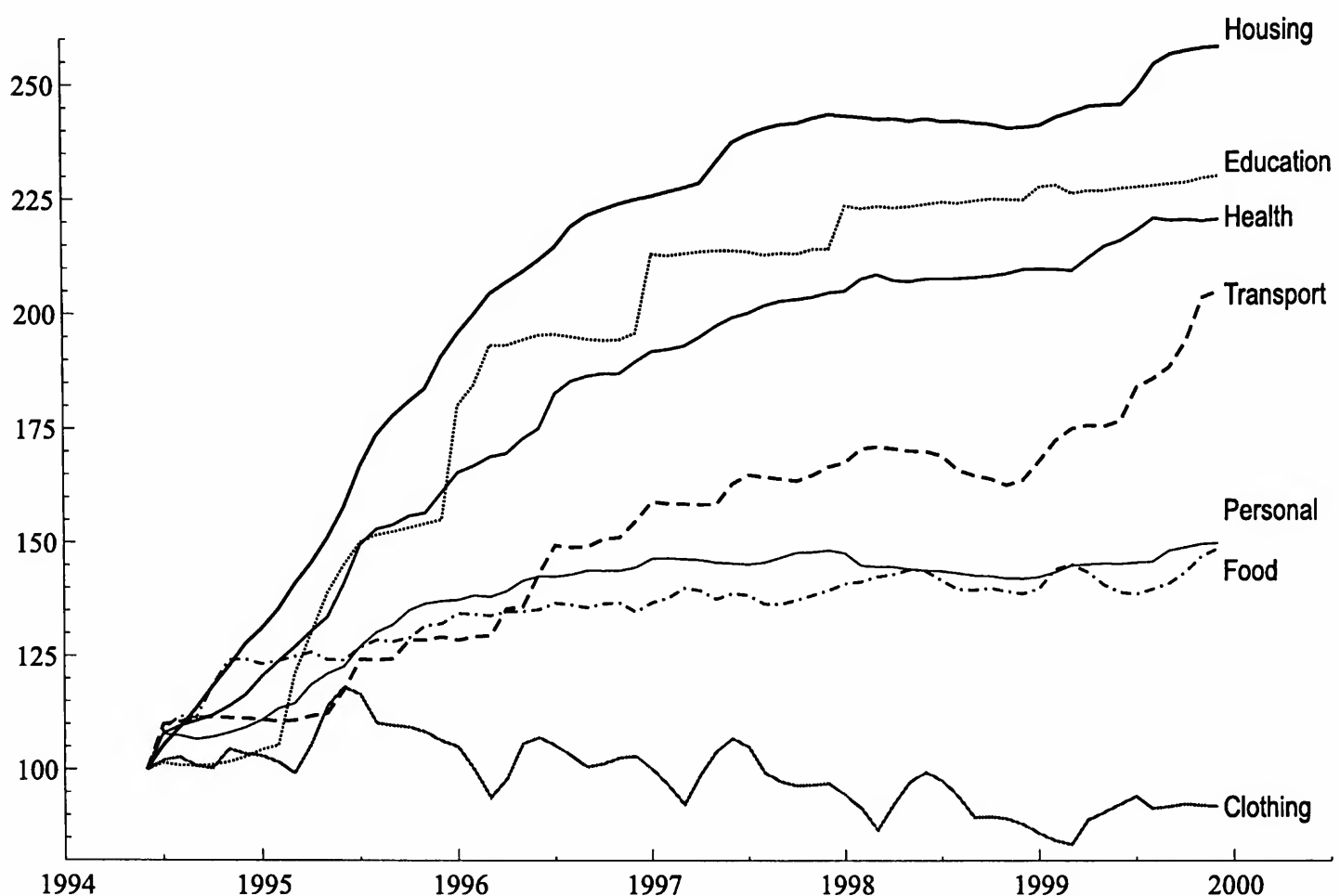
**Figure 2**



As can be seen in the above figure, the behavior of the series in this particular time span seems to reflect a consistent combination of economic policy designed to tame inflation to reasonable levels (with negative rates of variation during some periods).

For the purpose of our core inflation measurement methodology below, we will analyse the behavior of the seven major components of IPC-FIPE: Food, Housing, Transportation, Personal Expenses, Clothing, Health and Education. All of the data used here, along with explanations regarding methodology in the index construction, as well as updates, can be retrieved in the site [www.fipe.com](http://www.fipe.com). Figure 3 below depicts the behavior of the seven indices for the major components:

Figure 3

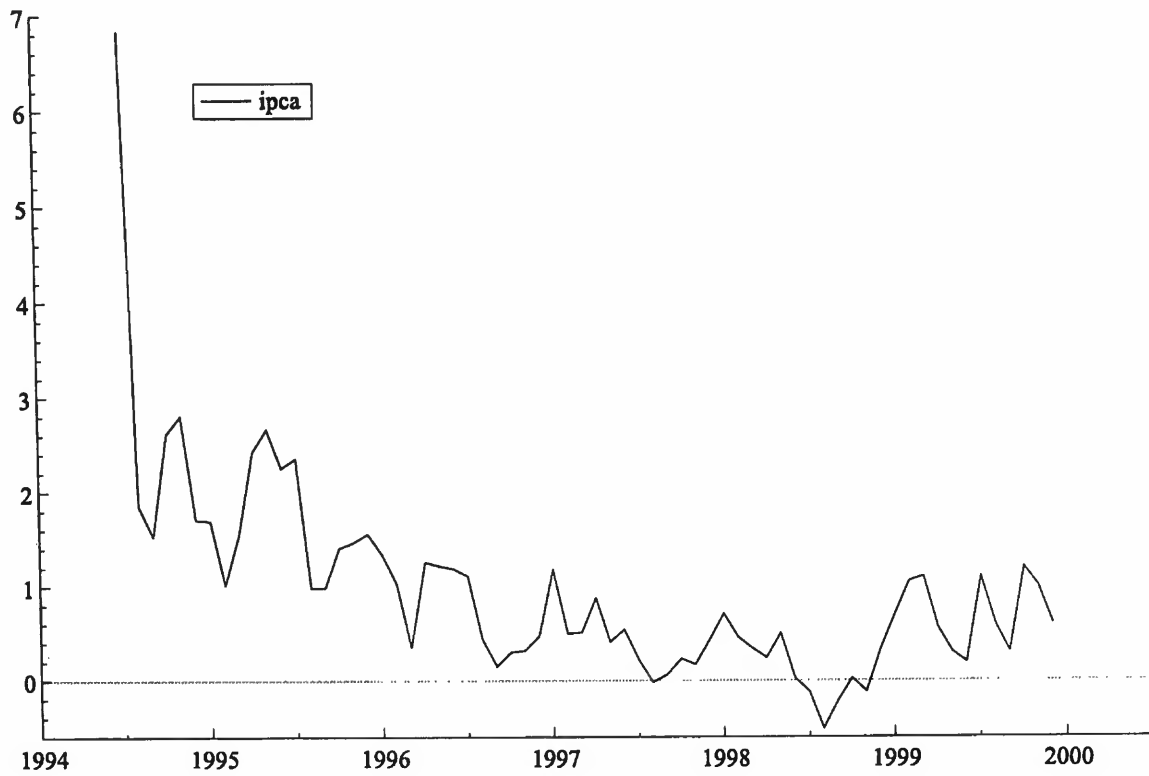


The behavior of the series shows a considerable variance during this period. It is interesting to note at this time that some of the series are clearly non-stationary, whereas others are not. One of the main statistical advantages in using the Kalman Filter for extracting a common stochastic component of these series, as done below, is that there is no need to relate this long-term concept to the usual idea of equilibrium contained cointegration relations, which would be troublesome in this case. One striking feature from Figure 3 is the presence of administered prices, such as the ones in the health group, which evolve in a pattern which would not conform to our interpretation of prices formed freely in markets, subject to supply shocks. This interesting point was made by one of the referees, and subsequent versions of our study will aim to deal with this fact directly.

## 2.2 IPCA-IBGE

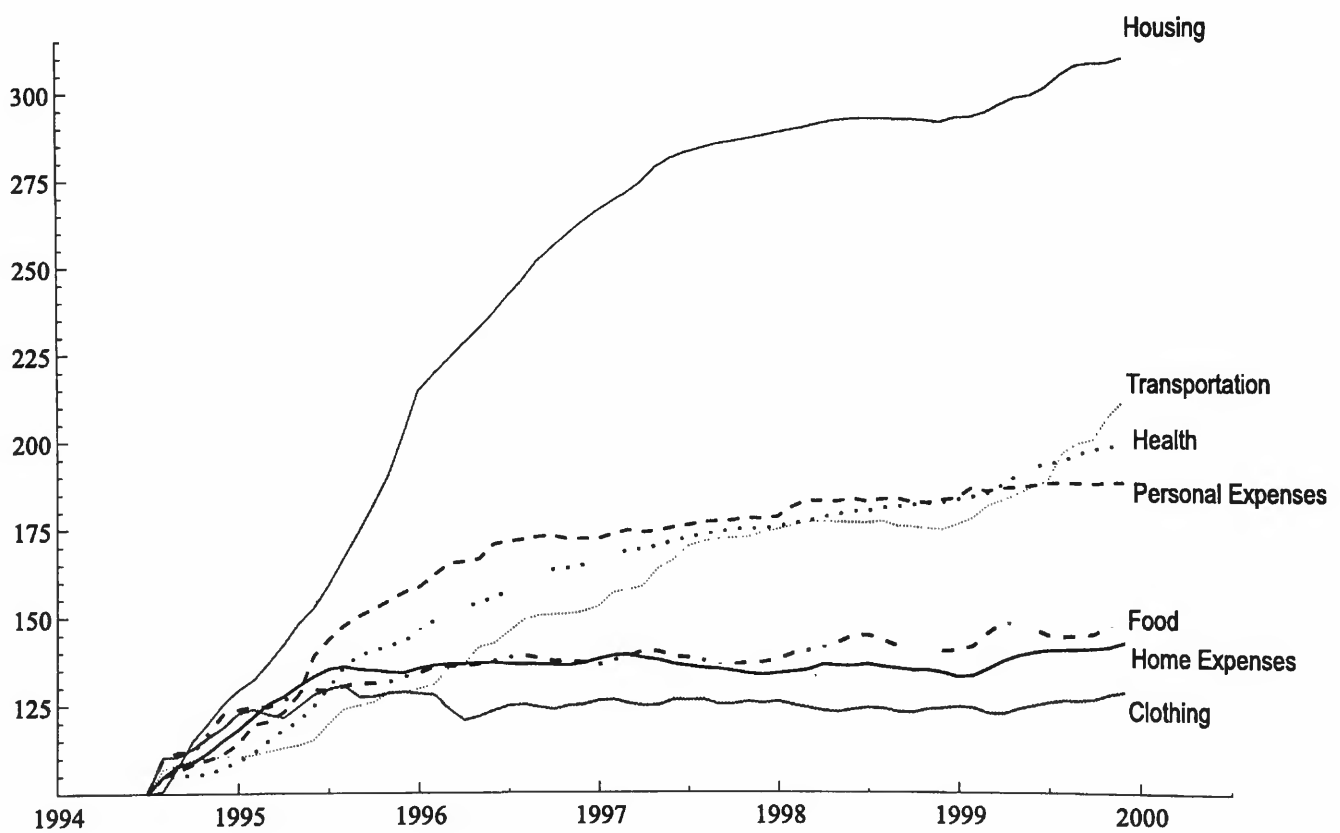
This index is calculated monthly by IBGE, and contrary to the IPC-FIPE it has a nationwide coverage. Figure 4 below depicts the behavior of the variation for the IPCA between July 1994 and December 1999:

**Figure 4**



The IPCA is also decomposed into seven components. Figure 5 below depicts the behavior of the indices for each component across the sample period:

**Figure 5**



The same point on the usefulness of the Kalman Filter methodology which was made for the IPC-FIPE is clearly also relevant here.

### 3 The common stochastic component model

Here we address the question of defining a measure of core inflation, which can be thought in intuitive terms as a measure of the permanent and fundamental shocks to the general price level, as stated in Section 1. We propose to do so by estimating a dynamic factor model in first differences of the seven components of the IPC-FIPE and the IPCA-IBGE explained in the last section, following Stock and Watson's (1991) methodology for constructing a Coincident Indicator for the US economy. The model is based on the relationship between each series and a common component:

$$\Delta Y_{it} = D_i + \gamma_i \Delta C_t + e_{it}; \quad i = 1, \dots, 7 \quad (1)$$

The index  $t$  points to each period in the sample, whereas the index  $i$  here selects each of the seven components of the IPC-FIPE index, whose differences are represented as  $\Delta Y$ . The common (unobserved) component of each of these series is represented in first difference by  $\Delta C$ , and is related to each of the four series via a specific weight given by  $\gamma_i$ , which will be estimated here along with the other parameters. In addition, the behavior of each of the seven series is determined by an individual component given by  $D_i + e_{it}$ , more of which below. Equation (1) will be directly interpreted as the transition equation in the state-space formulation of the model. The stochastic terms of the individual components may be formulated so as to incorporate a dynamic effect from shocks as:

$$e_{it} = \theta_{i1} e_{i,t-1} + \theta_{i2} e_{i,t-2} + \dots + \theta_{iq} e_{i,t-q} + \varepsilon_{it} \quad (2)$$

where  $\varepsilon_{it} \sim NID(0, \sigma_i^2)$ ;  $i = 1, \dots, 7$ . The Transition Equation for the state-space formulation can be represented as

$$\Delta C_t - \delta = \phi_1 (\Delta C_{t-1} - \delta) + \phi_2 (\Delta C_{t-2} - \delta) + \dots + \phi_p (\Delta C_{t-p} - \delta) + u_t \quad (3)$$

where  $u_t \sim NID(0, \sigma_u^2)$ .

In order for the parameters of (1)-(3) to be estimated, we set the Transition Equation as an Markovian process, so that we can apply the Kalman Filter in conjunction with maximum likelihood to account for the unobserved components. In selecting a particular specification, we followed the Schwarz Information Criterion, which penalizes the likelihood for the inclusion of unnecessary parameters. The final specification chose  $p = q = 1$  for the eighth equations, written in deviations from means. Therefore, in matrix form we can represent the Measurement Equation as

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \vdots \\ \Delta y_{7t} \end{bmatrix} = \begin{bmatrix} \gamma_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta c_t \\ e_{1t} \\ \vdots \\ e_{7t} \end{bmatrix} \quad (4)$$

or simply  $\Delta y_t = H\alpha_t$ . The Transition Equation as

$$\begin{bmatrix} \Delta c_t \\ e_{1t} \\ e_{2t} \\ \vdots \\ e_{7t} \end{bmatrix} = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \theta_{11} & 0 & 0 & \dots & 0 \\ 0 & 0 & \theta_{12} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & \theta_{71} \end{bmatrix} \begin{bmatrix} \Delta c_{t-1} \\ e_{1,t-1} \\ e_{2,t-1} \\ \vdots \\ e_{7,t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{7t} \end{bmatrix} \quad (5)$$

or simply  $\alpha_t = T\alpha_{t-1} + v_t$ . For a good explanation on the estimation of the parameters in general state-space model formulation, the reader is referred to Harvey (1989).

#### 4 Estimation results

The output from the maximum-likelihood estimation of the parameters from the space-state model of the previous sections is:



Parameter	IPC-FIPE		IPCA-IBGE	
	Value	Standard Error	Value	Standard Error
$\phi_1$	0.67029	0.12422	0.83806	0.10157
$\theta_{11}$	0.42415	0.12143	0.66726	0.11218
$\theta_{12}$	0.92367	0.0568	0.98192	0.021215
$\theta_{13}$	0.30173	0.13009	0.77832	0.094201
$\theta_{14}$	0.19920	0.14048	0.59396	0.12850
$\theta_{15}$	0.34483	0.11444	0.30598	0.13374
$\theta_{16}$	0.45208	0.23447	0.58844	0.15415
$\theta_{17}$	0.19754	0.12702	0.18957	0.16250
$\sigma_1$	2.0383	0.35932	1.1314	0.21475
$\sigma_2$	0.29162	0.085554	0.28950	0.11370
$\sigma_3$	2.4543	0.44073	0.32263	0.058594
$\sigma_4$	0.77663	0.14521	1.4988	0.28106
$\sigma_5$	9.5961	1.6722	1.1725	0.21209
$\sigma_6$	0.12212	0.083883	0.13859	0.031883
$\sigma_7$	7.2860	1.2755	0.81310	0.16246
$\gamma_1$	0.59318	0.18331	0.88540	0.16050
$\gamma_2$	0.22571	0.11072	-0.79482	0.11761
$\gamma_3$	0.97828	0.21344	0.33335	0.086939
$\gamma_4$	0.74069	0.11878	0.92290	0.18626
$\gamma_5$	0.33819	0.37489	0.48274	0.13269
$\gamma_6$	1.1761	0.11647	0.58050	0.071694
$\gamma_7$	0.74438	0.32317	0.74326	0.11948
Log-Likelihood:	-402.21		-190.49	

What can be seen in the above table is the overall statistical significance of the parameters. Even though this is not a standard estimation result, it is still obtained through maximum-likelihood, meaning that the asymptotic properties of the estimator are all here. The interpretation of these results is best seen through the graphical analysis of the calculated series for the unobserved common stochastic component  $c_t$  for the two indices. Figure 6 compares the general index for the IPC-FIPE with the calculated  $c_t$ , which we will name, as in Bryan and Cecchetti (1995), the dynamic factor index (DFI):

Figure 6

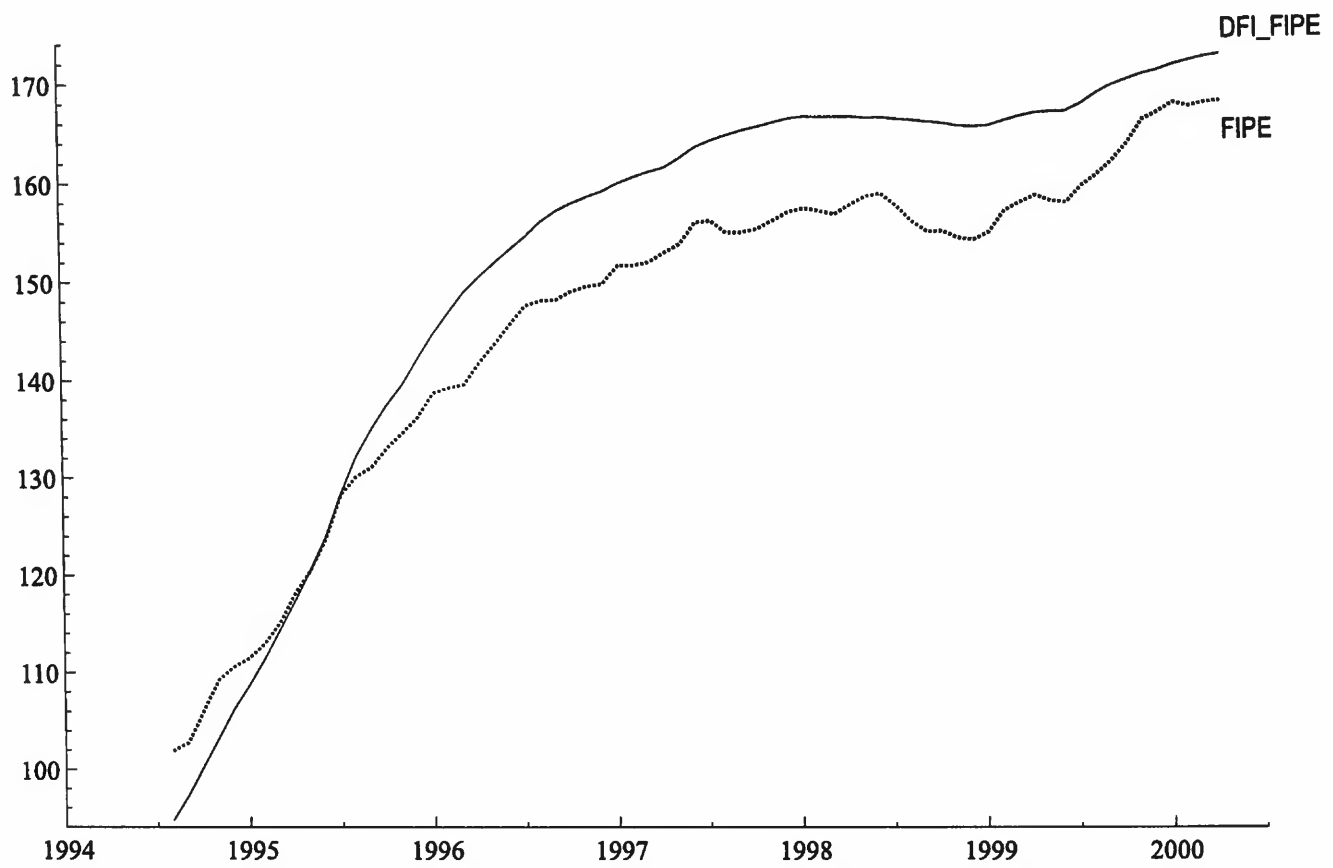
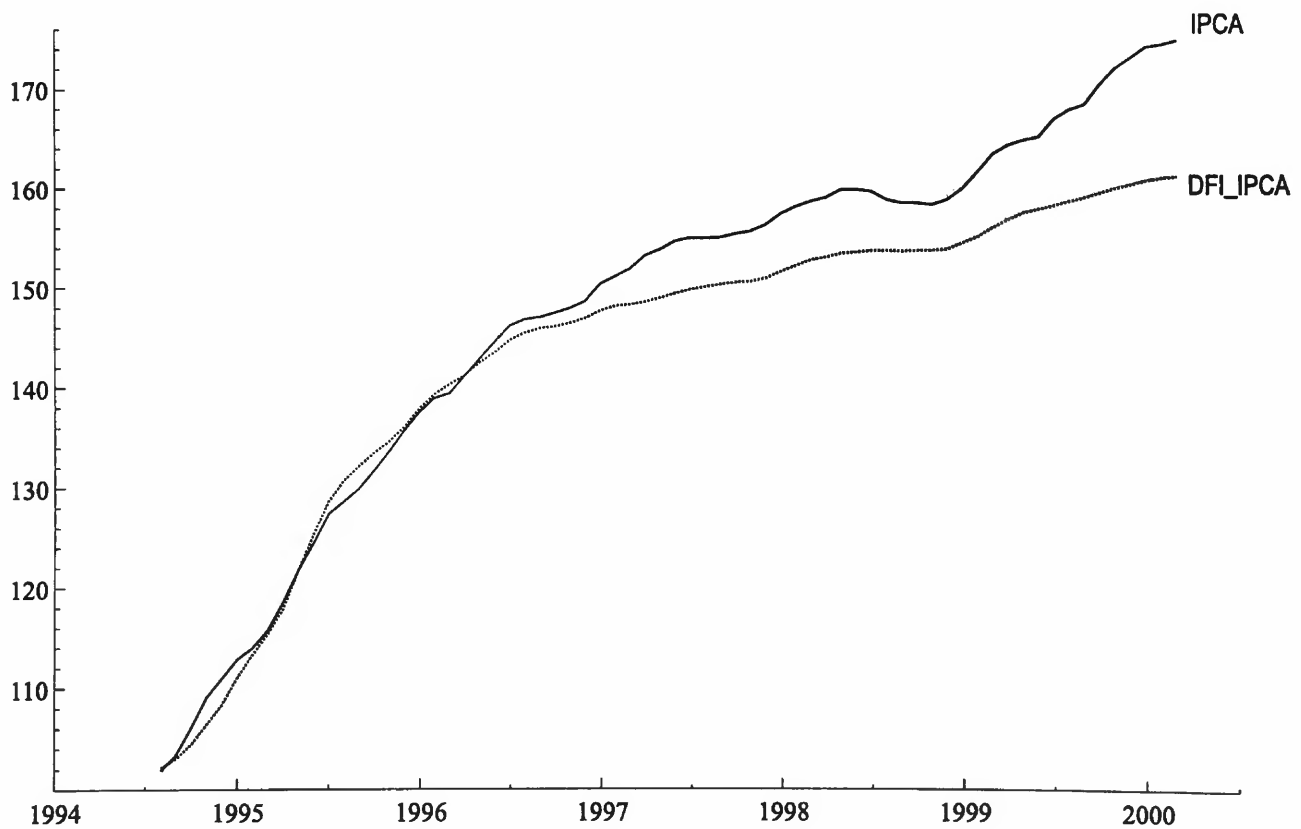


Figure 7 below compares the same two series, this time for the IPCA-IBGE:

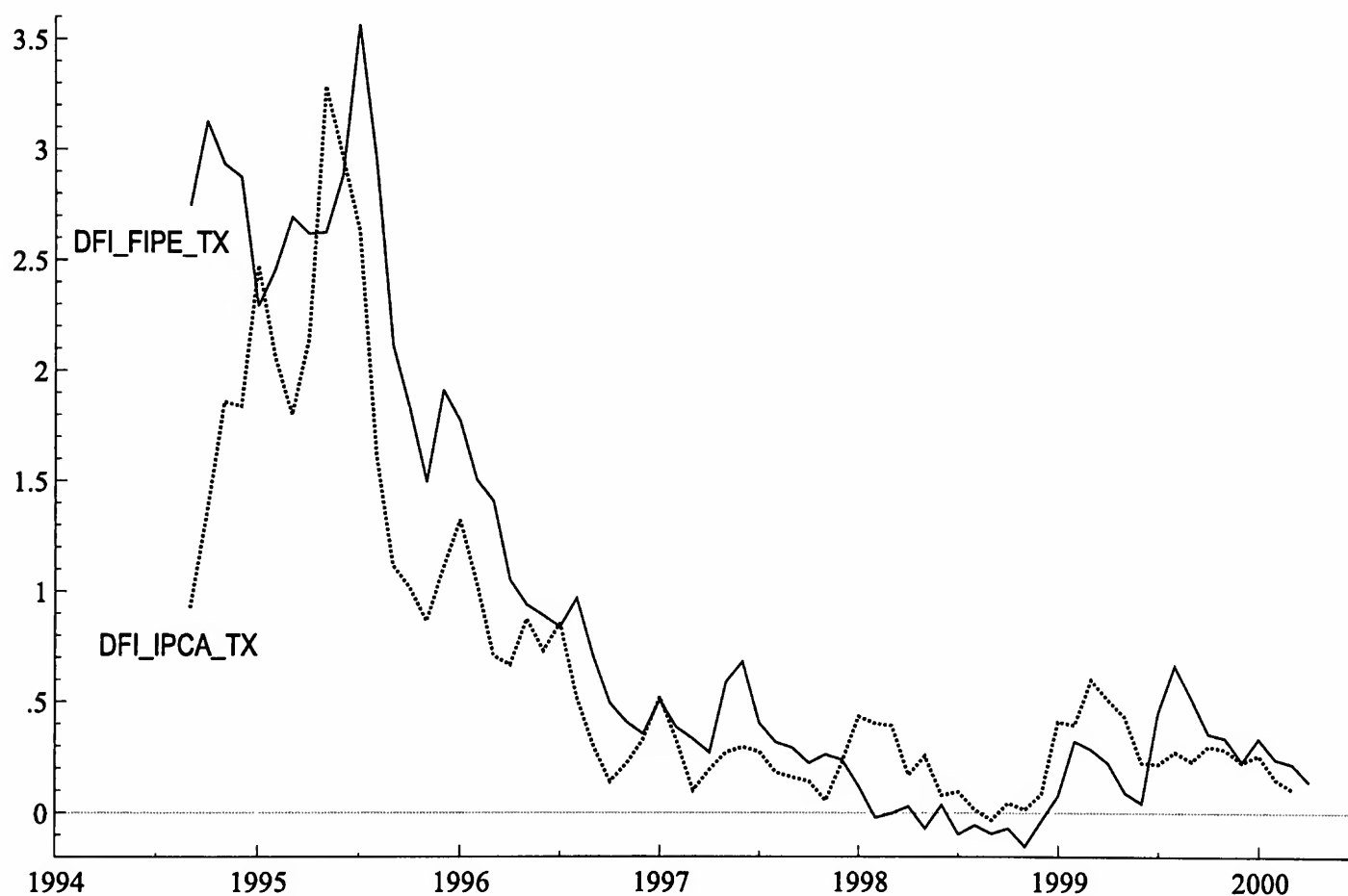
Figure 7



One important aspect in Figures 6 and 7 is that, whereas the dfi measures are above the respective indices for the period between mid 1995 and mid 1999, both of them fall below the indices after 1999. This indicates that both indices would be subject to largely transitory shocks beginning in the second semester of 1999, and as so trying to extrapolate a tendency after these indices would over-estimate the longer run “core” tendency of inflation, contrary to what would happen if one used the dfi measures instead. The behavior of both indices in the first months of 2000 seems to clearly indicate that the dfis would forecast the core trend of inflation substantially better.

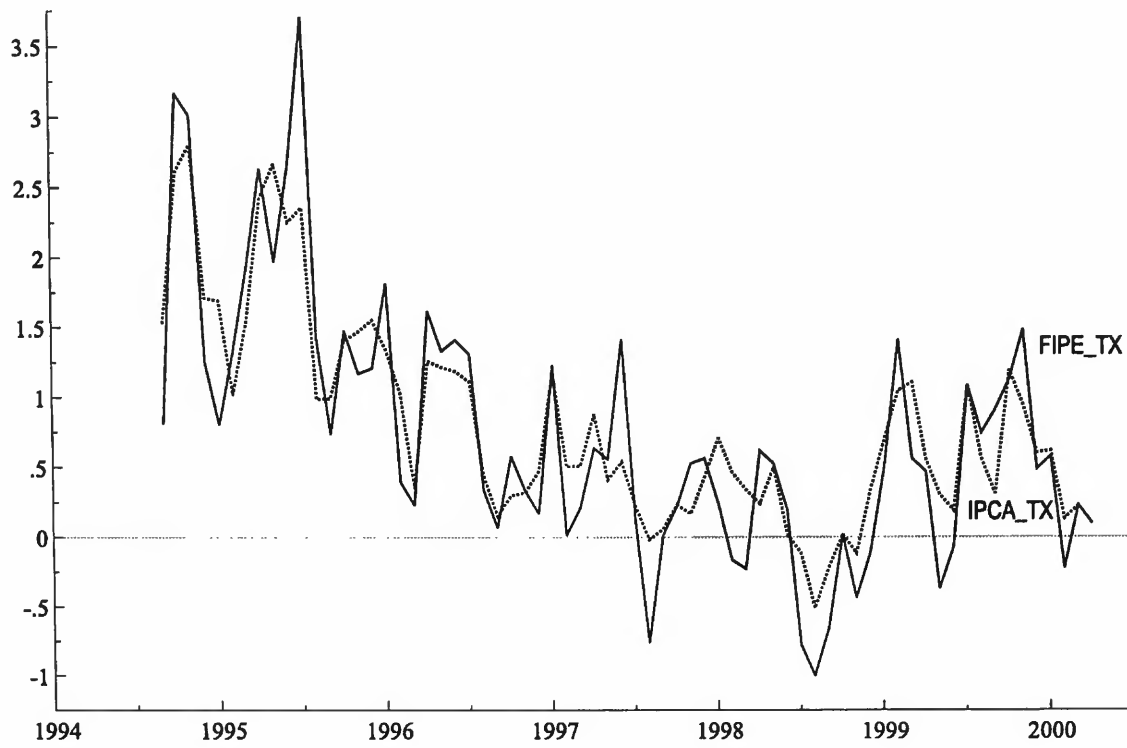
It is very interesting to compare the calculated rates of variations from the DFIs for the two indices:

**Figure 8**



and then compare the rates for the two actual indices:

**Figure 9**



Although the series of variations for the two indices are obviously highly correlated, this is even more so for the two calculated dfis. Figure 10 depicts the cross-correlograms between both rates and DFIs for the two indices:

**Figure 10**

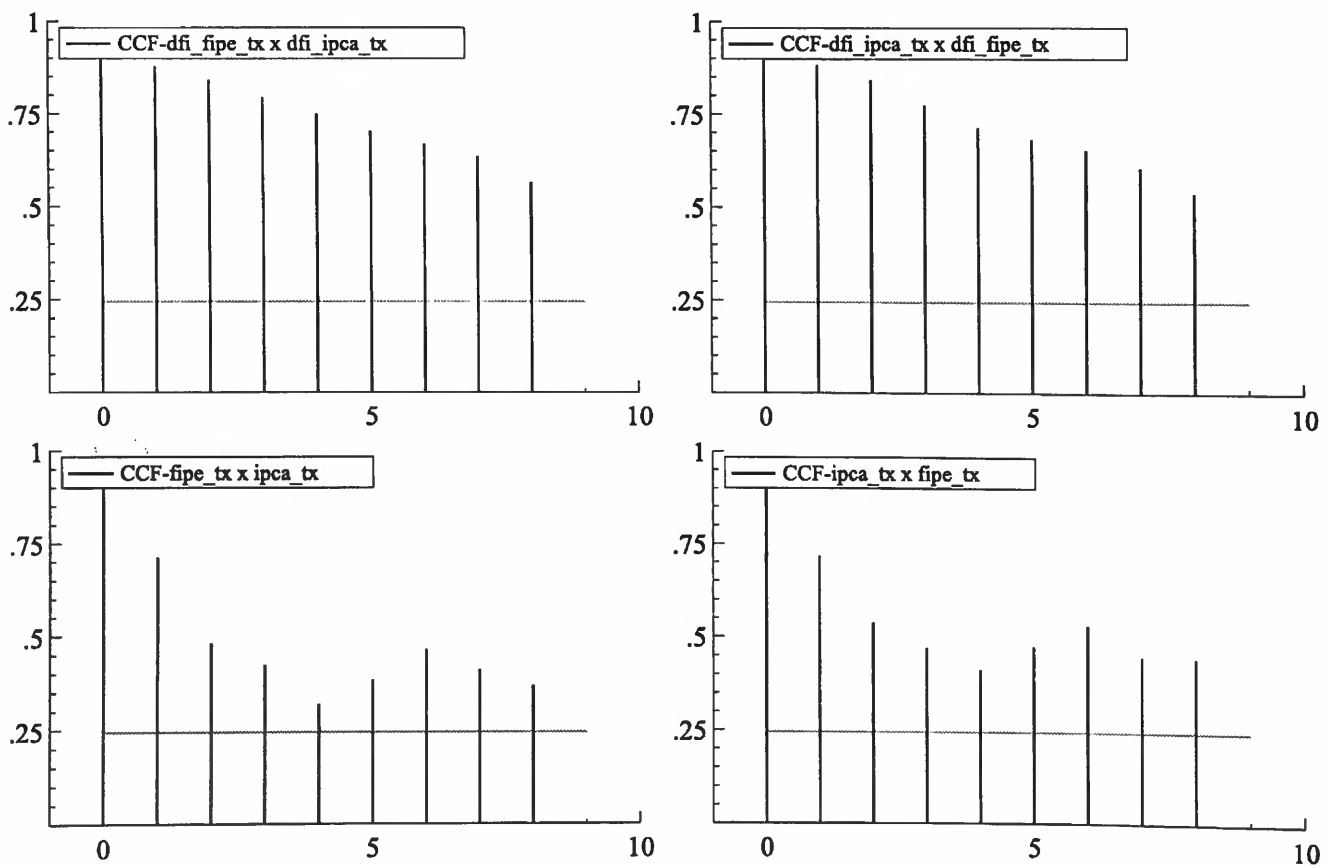
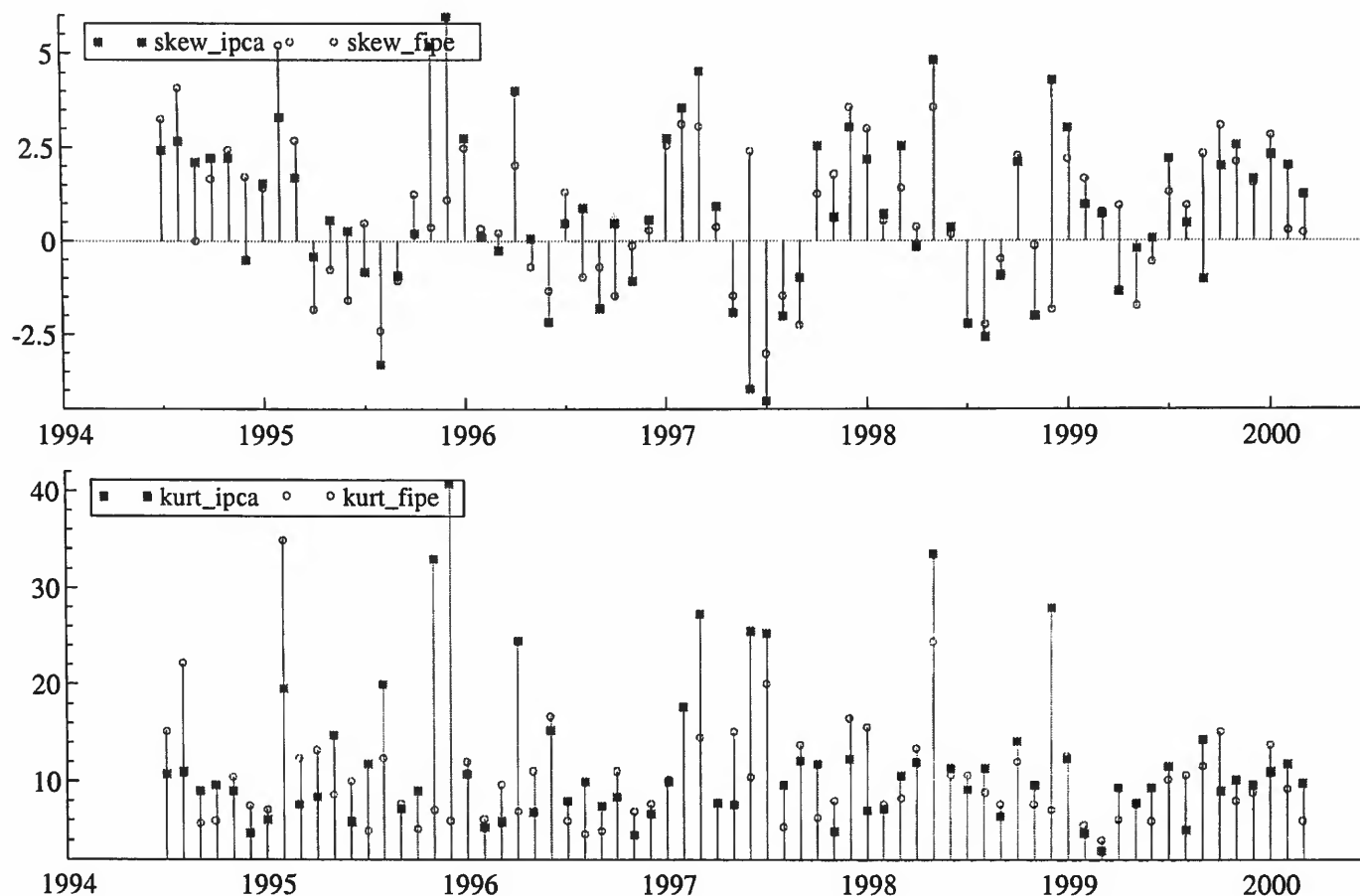


Figure 10 conveys the idea that the DFIs are doing a good job in capturing the fundamental factors behind the sustained price movements. Their cross-correlograms show a more definite pattern compared to the cross-correlograms of the rates of variations for the two indices, which are subject to specific price-shocks.

## 5 Theoretical justifications for trimmed means

Ball and Mankiw (1995) state conditions under which relative-price changes constitute supply shocks, which can alter general price indices momentarily, but not permanently, if there is no monetary accommodation. The main argument is based on a context where firms are subject to, say, shocks in their costs, but differ in their ability or willingness to adjust their own prices accordingly. The first category of price-setters adjust their prices instantly and continually in response to shocks in their own production costs. The second category of firms incur a cost for adjusting their own prices (menu-costs argument), and therefore do not change their own prices as often as the firms form the first category. This produces a smoother time path for the prices of this category of firms, reflecting their long-run expectations, which are based on zero-mean supply shocks on relative-prices, which are not accommodated by monetary policy, and therefore do not produce an long-run tendency for the general price level. The first category of firms, on the other hand, can be accounted for introducing the kind of noise in the time path of general price levels, which is exactly what the concept of core inflation intends to avoid. In this context, if the distribution of relative-price shocks is positively skewed, then more firms will be raising their prices than lowering them. However, the firms facing the menu costs and forming expectations about the future in a rational way will not choose to raise their prices. What this means is that the right tail of the distribution will be comprised mainly of the price raises of the first category of firms, which do not care about the long-run tendencies, given their ability to quickly and costlessly correct for mistakes in their pricing policies. Therefore, the highest raises in the cross-section of prices in each period represent, in this context, the transitory noise which we seek to eliminate. Bryan and Cecchetti (1994) relate this theoretical argument directly to the use of the statistical measure of core inflation based on trimmed means. Following the above argument, the core component of inflation would be given by the interior portion of the cross-section distributions of the price index, which would reflect primarily the price adjustments relative to the demand (monetary) shocks related to the effective long-run trend of inflation. Figure 11 below depicts the dynamic behavior of the cross-section skewness and kurtosis measures for both the IPCA-IBGE and the IPC-FIPE during our sample period:

Figure 11



The three striking factors are:

- 1 In general, the measures of symmetry and skewness for the two indices are highly correlated over time;
2. The distributions for the cross-section variations are highly leptocurtic;
3. The distributions for the cross-section variations are also generally positively-skewed.

The highly leptocurtic pattern can be taken as evidence in support of the theoretical considerations above. At the limit, price adjustments relative to demand/monetary shocks would converge to a spike for those who did not suffer supply shocks large enough to pay the adjustment costs of changing their prices accordingly. A simple regression provides some evidence on the relation between skewness and variations the IPCA:

$$IPCA_t = 0.8522 IPCA_{t-1} + 0.06705 IPCA\_SKEW_t + \text{seasonals}$$

(0.06446)                      (0.02728)

$$R^2 = 0.912246.$$

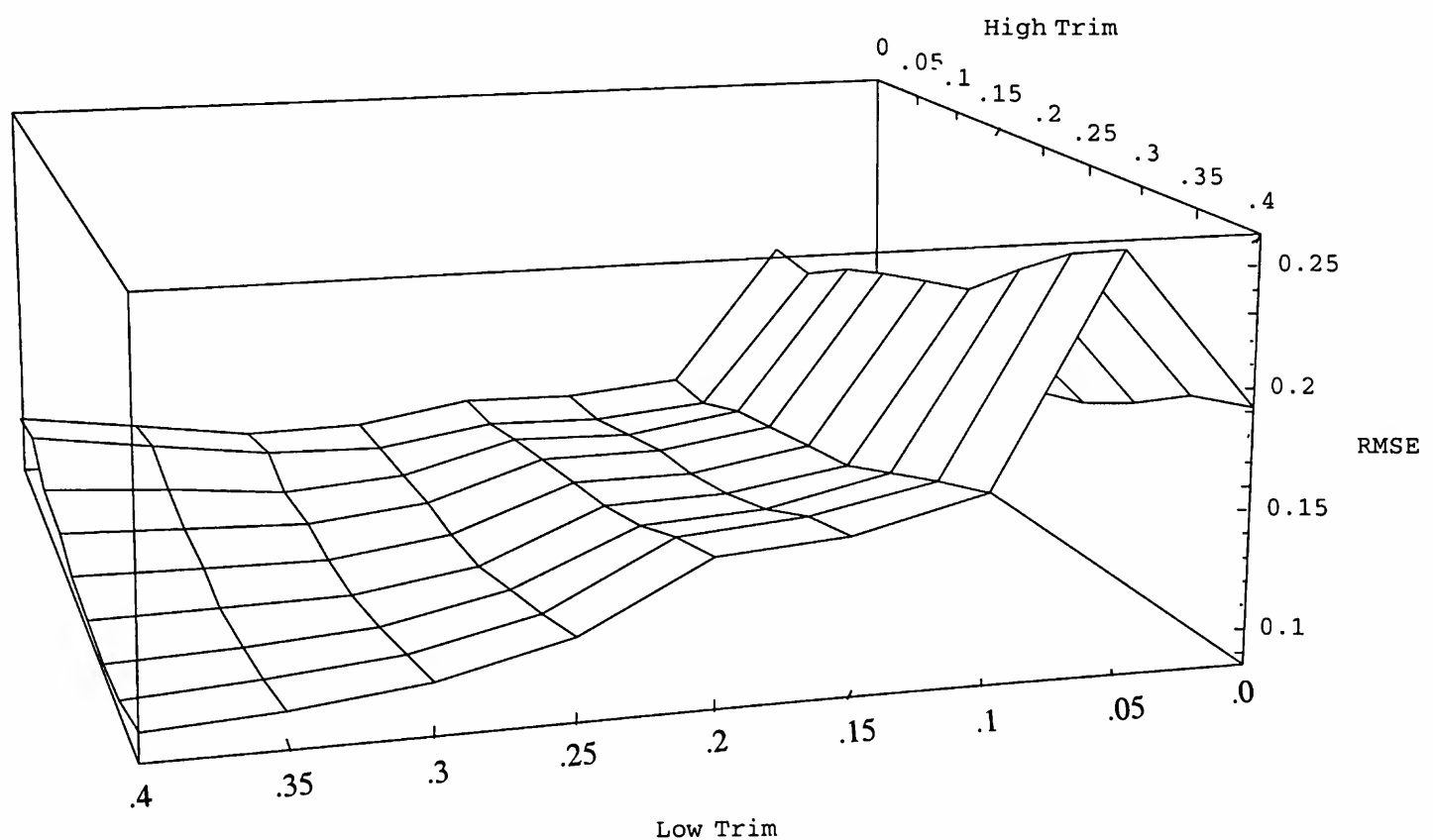
## 6 Trimmed means calculations

The results for the third and fourth moments of the cross-sections for the IPCA-IBGE seen in last section provide a justification for the use of what Bryan and Cecchetti (1994) name as “limited influence” estimators for the core inflation concept. The two limited influence estimators are the trimmed mean, and the weighted median. We will not review these concepts here, preferring to refer the interested reader to the excellent survey by Laflèche (1999). Even though the trimmed mean concept is supported on theoretical and statistical grounds, there is no clear consensus on the amount of the trim. Our approach here is to consider several different combinations of trims in the lower and higher ends of the cross-sectional distributions for the disaggregated items of the IPCA-IBGE and the IPC-FIPE, and then compare the performance of these combinations in terms of root-mean squared error (RMSE) with respect to the respective calculated DFIs.

### 6.1 IPCA-IBGE

Figure 12 compares the behavior of the choices of trims in terms of RMSE to the IPCA-IBGE calculated DFI:

**Figure 12**

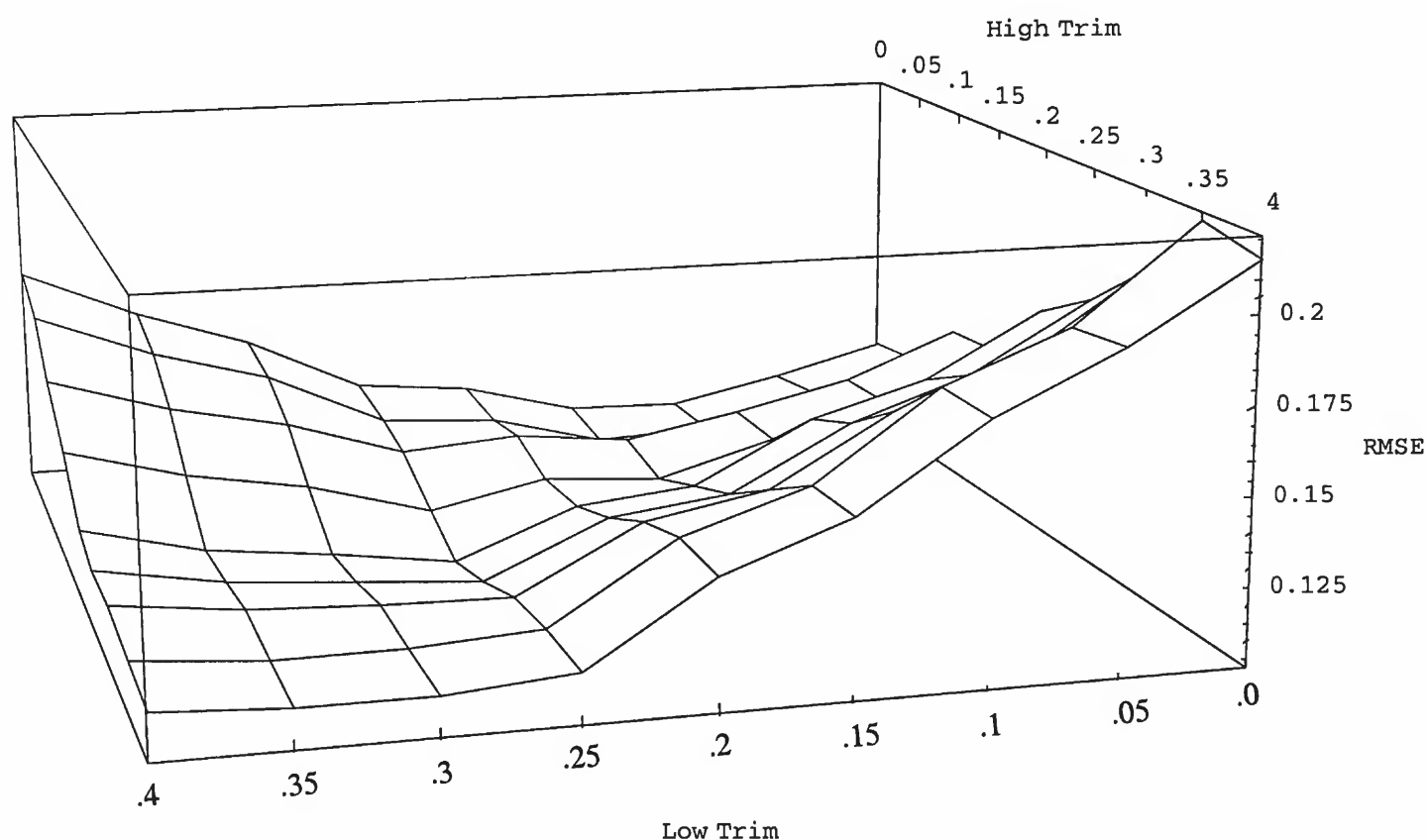


The optimal combination is 30% for the inferior trim, and 40% for the superior trim, which produces a RMSE of 0.720338. The RMSE for the weighted median, the limiting case in this trimming strategy, is 0.76282. Examining Figure 12 we can see that the RMSE is very sensitive to the choices of trims.

## 6.2 IPC-FIPE

Figure 13 compares the behavior of the choices of trims in terms of RMSE to the IPC-FIPE calculated DFI:

**Figure 13**



The optimal combination is 20% for the inferior trim, and 25% for the superior trim, which produces a RMSE of 0.8527. The RMSE for the weighted median is 1.0470. It is interesting to see that apparently, for the IPC-FIPE, the choices of trims produce reasonably stable values for the RMSE, as long as the trims are roughly simmetrical.

Both in the IPC-FIPE and the IPCA-IBCE cases, the choice of the optimal trims is also sensitive to the sample period used in the RMSE calculation. The first observations of our sample, as can be seen from Figures 2 and 4, are considerably higher in value than the rest of the sample, and also produce a high influence on the choice of the optimal trims. Conducting the

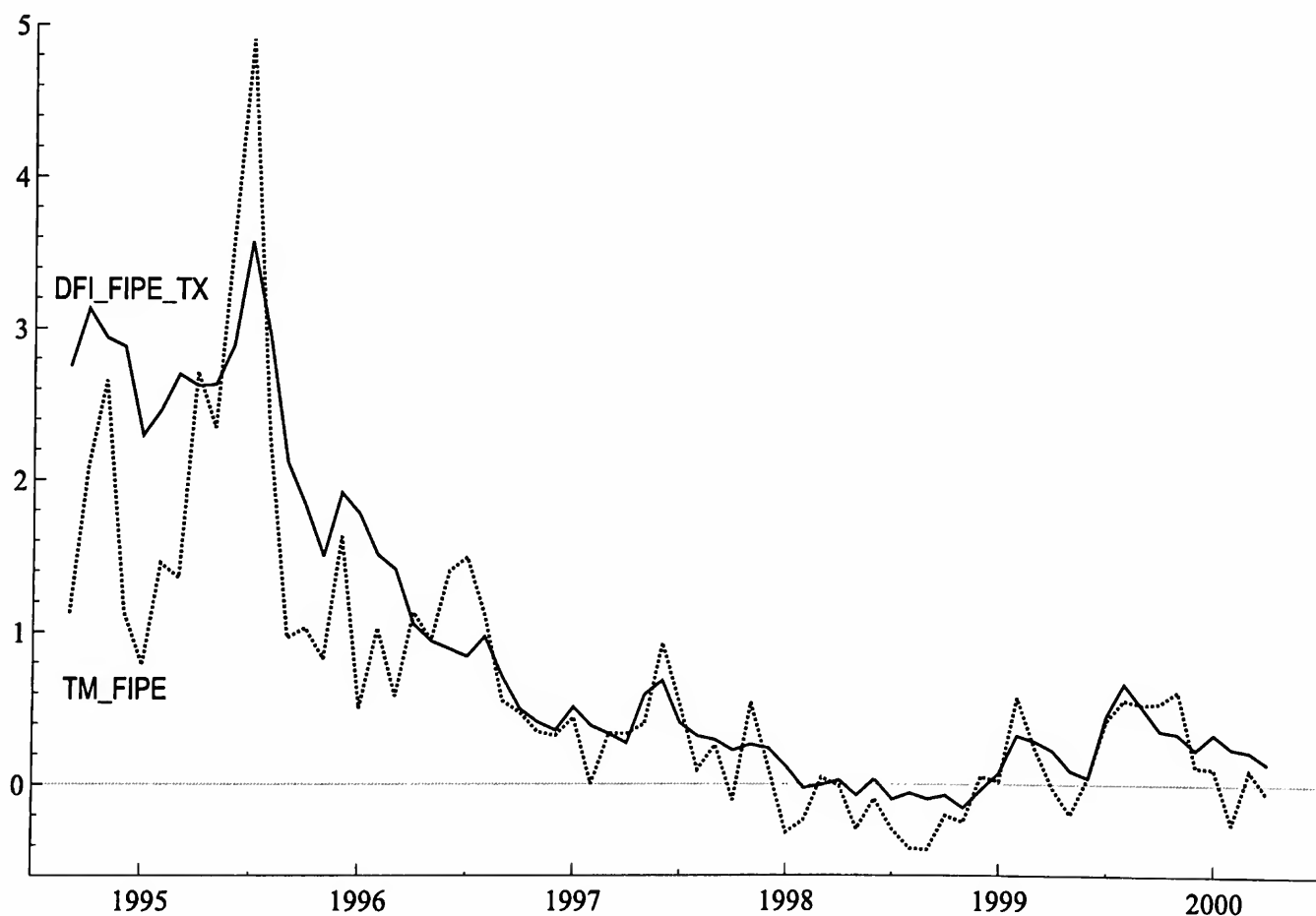


same RMSE exercise above, but discarding these observations, produce different values for the optimal trims, which indicates that the optimal trims do not seem to be stable over time.

## 7 Interpretation of results and conclusions

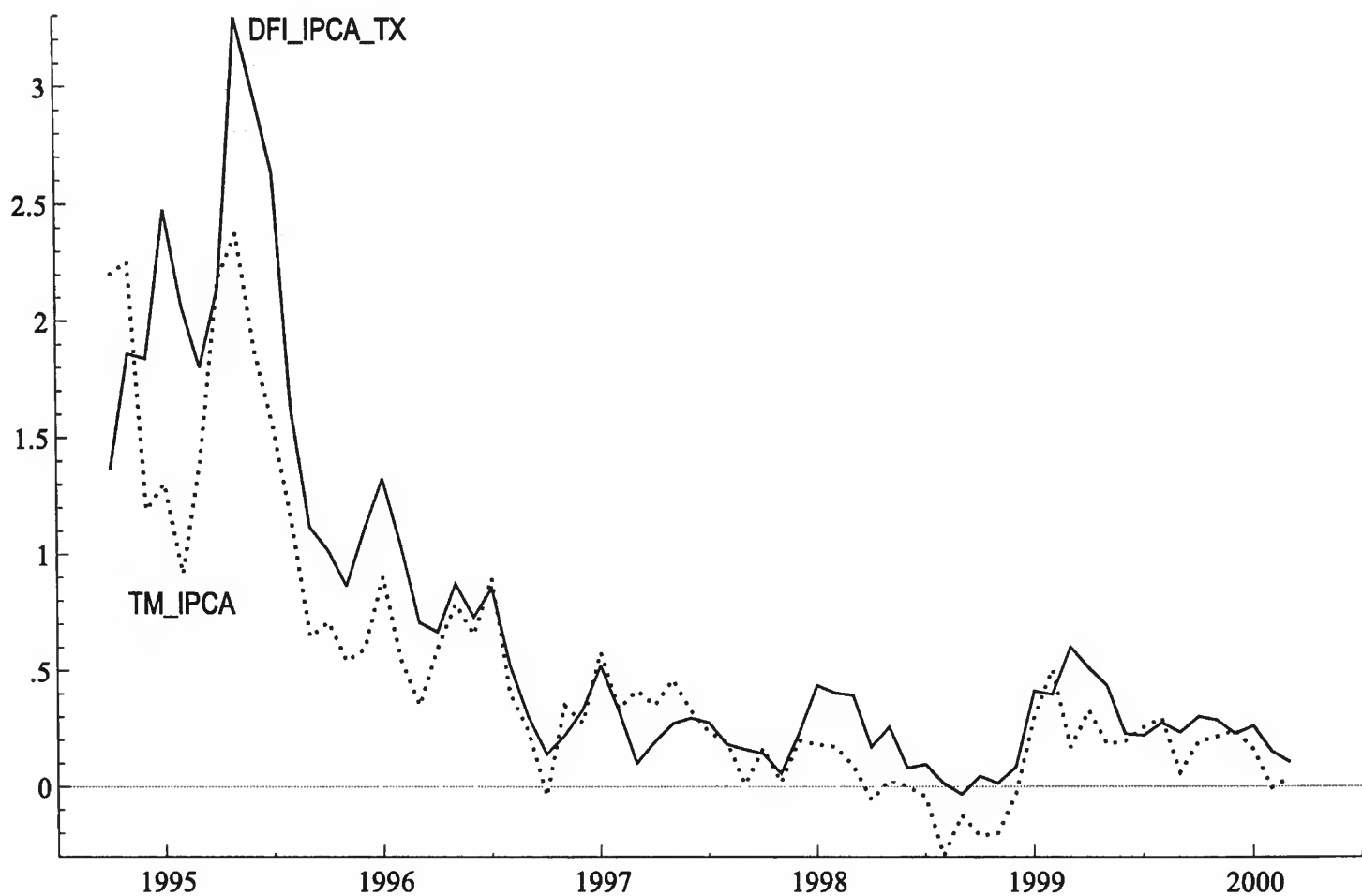
“Limited-influence” estimators of core inflation provide an appealing methodology for the estimation of the component of the price-level indices which can be related to demand shocks, and therefore the long-run tendency which we label inflation. However, there is no direct way to determine the optimal amount of trimming for this estimators. Our approach uses as a benchmark the dfi calculated by the state-space formulation to select the low and high trims that best approximate this measure. The advantages of this benchmark rest on solid statistical formulations. The disadvantages are the high computational costs, and the fact that the results are sensible to the chosen sample. Neither of these problems are present in the trimmed means estimators, which makes both approaches complementary in an interesting way. How well the “optimal trimmed means” estimators replicate the results from the DFIs? Figure 14 compares the estimated rates of variations for the IPC-FIPE:

Figure 14



The DFI measure is apparently much more stable than its counterpart trimmed mean. Figure 15 presents the same comparison, this time for the IPCA-113CE:

**Figure 15**



In general, the same comments hold. However, there is one important difference, which merits further research. The biggest discrepancy seems to occur around mid-1995 in both cases. But, whereas the DFI is inferior to the trimmed mean in this period for the IPC-FIPE, the opposite holds for the IPCA-IBGE. This should reflect a fundamental difference in the nature of the two indices, in terms of geographic and basket of goods/services coverage, which accounted for a very different reaction for the events in this period.

The advantages of taking the DFI as a benchmark rest on solid statistical formulations. The disadvantages are the high computational costs, and the fact that the results are sensible to the chosen sample. Neither of these problems are present in the trimmed means estimators, which makes both approaches complementary in an interesting way. The results of the Granger tests in the appendix seem to indicate that the DFI measures would indeed capture the long-run trend of the price level, given its dynamic relation to the rate of money growth. The “optimal” trimmed means for both indices apparently capture the same effect, although not as strongly as the re-

spective DFIs. The fact that both the IPCA and the IPC-FIPE fail to depict this dynamic relation would indicate the presence of strong supply shocks introducing “noise” to the long-run monetary relation between the M2 and prices. However, this conclusion would be hasty at this point, given that during most of our sample period (between mid-1994 and early-1998) Brazil adopted a fixed-exchange rate regime. Moreover, as mentioned above, our sample period is also characterized by changes in optimal demand for money in Brazil, given the adaptation to the successful stabilization plan for inflation. Both of these factors severely weaken the relation between monetary policy and prices. Therefore, the results of the Granger tests should not be necessarily taken as evidence, at this point, for favoring one measure of core inflation over another. However, as new observations under the present flexible exchange-rate regime become available, we believe that the methodology proposed here provides a fruitful research path. Also, future research will contemplate the issues of dealing with the presence of administered prices, as suggested by one of the referees, and also with the apparent non-stability of the optimal trims through time.

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## Appendix

### Granger Causality Tests

Lags	M2 to IPCA			IPCA to M2		
	F-Statistic	P-value	SIC	F-Statistic	P-value	SIC
1	0.14403	0.70564	1.242596	3.47464	0,06721	5.076723
2	0.04672	0.95439	1.351771	4.50607	0.01525*	5.061836
3	0.24546	0.86420	1.449654	3.02107	0.03750*	5.205144
4	0.33695	0.85179	1.592621	3.14695	0.02181*	5.150119
5	0.95497	0.45468	1.481736	2.30543	0.05991	5.228990
6	1.45084	0.21684	1.486753	2.37288	0.04466*	5.162994

Lags	M2 to DFI_IPCA			DFI_IPCA to M2		
	F-Statistic	P-value	SIC	F-Statistic	P-value	SIC
1	5.45956	0.02276*	0.393132	2.45252	0.12251	5.153056
2	3.55782	0.03488*	0.440634	1.77097	0.17925	5.183504
3	2.26999	0.09055	0.514383	1.19222	0.32125	5.320900
4	4.67965	0.0066*	0.537285	1.00267	0.41464	5.439464
5	6.11681	0.00018**	0.200942	2.26299	0.06265	5.333864
6	6.13365	8.8E-05*	0.285632	1.76612	0.12729	5.478616

Lags	M2 to T_MEAN_IPCA			T_MEAN_IPCA to M2		
	F-Statistic	P-Value	SIC	F-Statistic	P-Value	SIC
1	2.14933	0.14777	0.694504	1.48826	0.22718	5.149408
2	0.34807	0.70751	0.396816	1.14766	0.32448	5.177447
3	0.55841	0.64473	0.506713	1.68010	0.18189	5.271936
-4	1.72123	0.15935	0.412644	1.45943	0.22794	5.367810
5	2.66589	0.03292*	0.239631	2.15089	0.07490	5.286241
6	2.13229	0.06749	0.371667	2.03742	0.07964	5.379122

\*Rejection of  $H_0$  at 5%

\*\*Rejection of  $H_0$  at 1%

SIC: Schwarz Information Criteria

Lags	M2 to FIPE			FIPE to M2		
	F-Statistic	P-value	SIC	F-Statistic	P-value	SIC
1	0.02601	0.87242	3.415656	3.58818	0.06301	5.041232
2	0.12173	0.88562	2.926832	3.49461	0.03702	5.062506
3	0.97590	0.41095	3.039602	2.38375	0.07936	5.160032
4	0.76633	0.55211	3.106514	2.25537	0.07593	5.290720
5	0.68084	0.64011	3.261051	1.85359	0.12026	5.273480
6	0.73751	0.62214	3.271975	2.65462	0.02729	5.395725

Lags	M2 to DFI_FIPE			DFI_FIPE to M2		
	F-Statistic	P-value	SIC	F-Statistic	P-Value	SIC
1	0.72177	0.39888	0.051274	0.48768	0.48762	5.187591
2	3.15282	0.05014	0.040166	0.27934	0.75729	5.235420
3	2.68869	0.05519	0.122309	0.30182	0.82394	5.378962
4	2.55514	0.04962*	0.184113	1.38853	0.25078	5.408684
5	3.15593	0.01509*	0.223206	2.88687	0.02314*	5.285472
6	1.96249	0.09073	0.323462	2.22015	0.05787	5.355671

Lags	M2 to T_MEAN_FIPE			T_MEAN_FIPE to M2		
	F-Statistic	P-value	SIC	F-Statistic	P-value	SIC
1	6.20376	0.01549*	1.913899	4.01410	0.04957*	5.149861
2	2.55996	0.08.602	2.015144	3.80387	0.02804*	5.126740
3	2.00695	0.12364	2.106577	3.16702	0.03146*	5.240583
4	2.47823	0.05529	2.160776	2.62327	0.04509*	5.327453
5	2.30105	0.05896	2.254764	1.88251	0.11451	5.365245
6	1.52017	0.19287	2.406512	1.49086	0.20248	5.428606

\*Rejection of  $H_0$  at 5%

\*\*Rejection of  $H_0$  at 1%

SIC: Schwarz Information Criteria