

The applied perspective for seasonal cointegration testing: a supplementary note

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RESUMO

O objetivo dessa nota é suplementar o artigo recentemente publicado por Oliveira e Picchetti (1997). No mencionado trabalho, os autores fazem uma resenha dos procedimentos disponíveis para testar e estimar as relações de cointegração nas frequências sazonais. Essa resenha inclui, também, um teste para estabelecer a presença de raízes unitárias nessas frequências. Contudo, considerações relativas ao poder desse teste recomendam a utilização de outros com diferente hipótese nula. Nessa nota, discute-se o teste de multiplicador de Lagrange de Canova e Hansen, disponível na literatura desde 1995, que apresenta boas propriedades.

Palavras-chave: raízes unitárias sazonais, sazonalidade determinista, sazonalidade estocástica.

ABSTRACT

In a recent issue of this journal the available procedures for testing and estimating cointegration relationships at the seasonal frequencies are surveyed. There it is recognized that prior knowledge about the presence of particular seasonal unit-roots is necessary, and two tests that provide this preliminary information are also surveyed, leaving out of this set a third test which is likely to be one with the highest power. This note tries to supplement the aforementioned paper by presenting the properties of the alternative (supposedly more powerful) test.

Key words: seasonal unit roots, deterministic seasonality, stochastic seasonality.

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1 Introduction

The objective of this note is to supplement the paper by Oliveira and Picchetti (1997) which surveys not only available tests and procedures necessary to estimate cointegration relationships at the seasonal frequencies but also the preliminary tests which indicate the presence of any seasonal unit root(s) and the corresponding frequency(ies). The motivation is the same one mentioned in the referred paper, that is to say, “*the lack of treatment of seasonal cointegration, even in the most recent books on cointegration.*” (Oliveira and Picchetti, 1997, page 263) The importance of the preliminary tests can hardly be overstated since it is clear that if two (or more) series do not have unit roots at corresponding frequencies, the possibility of cointegration does not exist. Furthermore, testing for unit roots is not straightforward. Several difficulties arise, as will be discussed in the presentation of various tests that can be used to detect which unit roots are present and, specially, the Canova-Hansen (CH) tests¹ not discussed by Oliveira and Picchetti.

The organization of the note is as follows: in the second section a brief discussion of seasonal analysis of economic time series is provided; Section 3 mentions several tests dealing with unit roots detection at various frequencies. Section 4 surveys the CH tests, available in the literature since 1995, highlighting their nature and properties, and Section 5 concludes.

2 Seasonality in economic time series

The study of seasonal variations has a long history in the analysis of economic time series. (Hylleberg, 1992) Until not long ago, seasonal features of economic time series were viewed as a nuisance void of inherent economic interest. An illuminating citation which shows this point of view occurs in Hylleberg (1994), where William Stanley Jevons is quoted from an 1862 paper expressing this kind of opinion. Since those times until recently, the usual practice continued to be to focus on seasonally adjusted data, at least in the field of macroeconomics. Typically, this result is achieved by applying specific seasonal adjustment filters. This view dominated applied time series econometrics until its drawbacks, as well as the possible economic relevance of seasonality, have been fully recognized.

1 Canova and Hansen (1995).

Starting in the 1980's many economists have realized that the supposedly seasonal noise part of a series may contain important information about the non-seasonal component and, in addition, that the seasonal component of one series may contain information about the seasonal and non-seasonal components of other series. In other words, they realized that it is a mistake to attempt to decompose the world of economics into two mutually independent worlds, one being non-seasonal and of economic interest and the other one being seasonal and uninteresting. A first by-product of this interest was the effort to properly define 'seasonality'. Granger (1978) put forward a definition based on the concept of spectral density while Hylleberg (1986) was responsible for a less formal definition that, after some years, evolved into the following one: "*Seasonality is the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decision, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by the endowments, the expectations and preferences of the agents, and the production techniques available in the economy.*"(Hylleberg, 1992)

Several different time series models of seasonality are conceivable. For example:

1. seasonality can be modeled as deterministic, as done by Barsky and Miron (1989), or as periodic with unchanged periodicity, as did Hansen and Sargent (1993). The power spectra of these models have spikes only at the seasonal frequencies;
2. it can also be modeled as a sum of a deterministic process and a stationary stochastic process.(Canova, 1992) In cases 1. and 2. the phenomenon can be conventionally modeled using seasonal dummies that allow some variation but no persistent change in the seasonal pattern over time;
3. another approach is to model seasonal patterns as non-stationary by allowing for (or imposing) seasonal unit roots, as suggested by Box and Jenkins (1976). These non-stationary seasonal processes show a varying and changing seasonal pattern over time, which cannot be captured using deterministic seasonal dummies because the seasonal component drifts substantially over time. Instead, such a series needs to be seasonally differenced to achieve stationarity.

The apparent variety of available models calls for simple statistical techniques that can discriminate between various forms of seasonality. In the next section a brief survey of some testing frameworks is presented.

3 A brief survey of unit root tests²

The detection of unit roots started to be studied in annual data (the so-called zero frequency). The extension of the resulting methodologies to consider seasonal frequencies occurred in two stages: first, the researchers studied the application to quarterly data - with the appearance of two additional frequencies -, and then considered monthly data which implies six seasonal frequencies in addition to the usual one. As soon as the new methods were known alternative procedures were proposed. In this way, not only parametric tests but also semiparametric, nonparametric and Bayesian techniques were put forward. For each one of them the three stage process was a natural development. Furthermore, in each case there were different proposals concerning the form of the null and alternative hypotheses, not to mention the large number of different data generating processes which were considered. The large number of possibilities implied by the combination of the above mentioned factors is still increased if we take into account different procedures, like those proposed by Dickey and Pantula (1987) concerning the order of differencing, and Ilmakunnas (1990) who proposes a two-step testing procedure. All this turns the complete survey of the area a Cyclops' task. Needless to say that such a complete survey goes beyond the scope of this note.

A rather lengthy debate between those who advocate the use of differenced data and those who prefer modeling original ('level') series (maybe with a trend variable) has been all but reconciled by the theory of cointegration of Engle and Granger (1987). A first-order integrated system is defined as an AR with roots outside the unit disc (or on the unit circle). It is assumed that the series can be made stationary by first-order differences. A similar development followed in the case of sub-annual data. In that case, and unless economic data have been seasonally adjusted by popular but frequently criticized routines such as Census X-11, they generally exhibit seasonal patterns which may be treated by including dummies in the system or modeled with additional unit roots at seasonal frequencies.

Non-adjusted monthly (or quarterly) economic time series showing seasonal patterns shed some doubts on the assumption of stationary first differences. The question whether these seasonal patterns should be eliminated by regression on seasonal dummies (the 'deterministic' model) or by treating them by seasonal differencing, thereby assuming additional unit roots on the unit circle (the 'stochastic' model), reminds of the discussion of deterministic and stochastic trend models. The existence of unit roots at the seasonal

2 For a discussion of the so-called 'roots of unity' (or unit roots) see Aguirre (1997).

frequencies has similar implications for the persistence of shocks as in the case of the existence of a unit root at the zero frequency. However, a seasonal pattern generated by a model characterized solely by unit roots seems unlikely as the seasonal pattern becomes too volatile, allowing ‘Summer to become Winter’

A process demanding filtering by $(I-B^s)$ - the so-called seasonal difference operator - in order to become stationary can be called a ‘seasonally integrated process’ an expression which does not have equivalent meaning when used by different authors. A naive procedure to handle such a process consists in the application of the $(1 + B + B^2 + \dots + B^{s-1})$ filter,³ which removes the intra-annual cycles but leaves the stochastic trend in the data. The seasonally adjusted series thus obtained is integrated of order one, and standard cointegration analysis can be applied to a vector consisting of these filtered series.

3.1 Unit roots at the zero frequency

The history of (non-seasonal) unit root tests starts with Dickey and Fuller (1979, 1981) and the well-known Augmented Dickey-Fuller (ADF) test. In this type of test the null hypothesis is a non-stationarity model (they suppose the existence of one or more unit roots).⁴ There are other tests with the same null, such as the CRDW-test based on the usual Durbin-Watson statistic (Sargan and Bhargava, 1983) and the nonparametric tests developed by Phillips and Perron, based on the Phillips (1987) Z-tests.

The discussion about the best choice of null hypothesis started more recently. Kahn and Ogaki (1992) present a test which has as null the statement that the series is stationary. The same objective is achieved with the KPSS test by Kwiatkowski, Phillips, Schmidt and Shin (1992) who also take a stationarity null against an alternative of existence of a unit root at the zero frequency. Since the Canova-Hansen tests are an extension of the latter, the KPSS test deserves a more detailed presentation.

KPSS’s starting point is the recognition that, in the early 1980s, most empirical studies showed that the majority of economic series contained a unit root. *“However, it is important to note that in this empirical work the unit root is the null hypothesis to be tested, and the*

3 This is the seasonal moving average operator.

4 For this reason the standard statistical techniques are not valid in that case. As a consequence, the tables with critical values for each test statistic must be generated via Monte Carlo simulations. The power of the ADF test in checking for the existence of a second (or third) unit root has been put in doubt by Dickey and Pantula (1987).

way in which classical hypothesis testing is carried out ensures that the null hypothesis is accepted unless there is strong evidence against it. Therefore, an alternative explanation for the common failure to reject a unit root is simply that most economic time series are not very informative about whether or not there is a unit root, or equivalently, that standard unit root tests are not very powerful against relevant alternatives.” (Kwiatkowski *et alii*, 1992, page 160) In the same journal article these authors give ample references of other papers which also provide evidence of the low power of Dickey-Fuller type tests.

Kwiatkowski *et alii* propose a test of the null hypothesis that an observable series is stationary around a deterministic trend, while the alternative states that the series is difference-stationary. They express the series as a sum of a deterministic trend, a random walk and a stationary error. In this way, theirs is a Lagrange multiplier (LM) test of the hypothesis that the random walk component of the series has zero variance. They present two test versions for the hypothesis of stationarity, either around a level or around a linear trend, and claim that the same methodology can be extended to allow for nonlinear trends.

Using the above parameterization and very restrictive assumptions about the nature of the random walk and the distribution of errors, the authors show that their LM test statistic is, under the null of trend stationarity, the same as other statistics obtained by other researchers using different approaches. However, considering that the assumption of white noise errors is too strong they relax it and study the asymptotic distribution of the statistics⁵ under more general conditions concerning the temporal dependence properties of the stationary error. As a result, they propose a modified version of the LM test statistic which is valid asymptotically under fairly general conditions.

Upper tail critical values for both statistics - calculated using simulations - are provided. The consistency of both tests is proved, and the finite sample size and power properties are studied by way of Monte Carlo experiments. The sizes of the tests depend only on sample size and the number of lags used to calculate the consistent estimate of the variance. While for some combinations of these parameters the asymptotic validity of the tests holds, for others considerable size distortions occur, specially when certain particular alternative hypothesis about the errors are considered. Concerning power, the authors show that only for sample sizes larger than 200 their tests are free of the trade-off between correct size and power.

5 The two statistics refer to the level-stationary and the trend-stationary cases.

Finally, Kwiatkowski *et alii* apply their tests to the same data used by Nelson and Plosser (1982) and find that, while they can reject the hypothesis of level stationarity for almost all of the 14 US annual macroeconomic time series under study, for many of them **they cannot reject the hypothesis of trend-stationarity**. These results agree with those contained in other studies and all of them support the existing doubts that most economic series contain a unit root despite the failure of Dickey-Fuller type - and other unit root tests - to reject the null hypothesis of difference-stationarity.

3.2 Unit roots at seasonal frequencies

The first test for seasonal integration resembles a generalization of the ADF test for integration in annual data. Dickey, Hasza and Fuller (1984), following the methodology suggested by Dickey and Fuller (1979) for the zero-frequency unit-root case, propose a test of the hypothesis $\rho = 1$ against the alternative $\rho < 1$ in the model $\gamma_t = \rho\gamma_{t-s} + \varepsilon_t$. The DHF test - as well as similar ones proposed in the following years - only allows for unit roots at all of the seasonal frequencies and has an alternative hypothesis which is considered rather restrictive, namely that all the roots have the same modulus. Trying to overcome these drawbacks Hylleberg *et alii* (1990) (from now on referred to as HEGY) propose a more general test strategy that allows for unit roots at some (or even all) of the seasonal frequencies as well as the zero frequency.

Since the HEGY test takes as null the existence of a unit root at one or more seasonal frequencies, *“rejection of their null hypothesis implies the strong result that the series has a stationary seasonal pattern. Due to the low power of the tests in moderate sample sizes, however, nonrejection of the null hypothesis unfortunately cannot be interpreted as evidence ‘for’ the presence of a seasonal unit root.”* (Canova and Hansen, 1995, page 237) Taking into account power considerations⁶ a useful complement to the above testing procedures would be any other test that takes *stationary* seasonality as the null hypothesis and the alternative to be *non-stationary* seasonality. *“In this context, rejection of the null hypothesis would imply the strong result that the data are indeed non-stationary, a conclusion that the DHF or HEGY tests cannot yield. Viewed jointly with these tests, such a procedure would allow researchers a more thorough analysis of their data.”* (Canova and Hansen, 1995, page 238)

6 Power is the probability of rejecting the null hypothesis in a statistical test when it is in fact false; the power of a test of a given null clearly depends on the particular alternative hypothesis it is being tested.

So, in the same way as HEGY generalizes the Dickey-Fuller framework from the zero frequency to the seasonal frequencies, Canova and Hansen generalize the KPSS approach from the zero frequency to the seasonal frequencies. They propose a test built on the null of unchanged seasonality which, in turn, can be adapted to test for unit roots at seasonal frequencies or for time variation in seasonal dummy variables. Despite the fact that the null under test is stationary seasonality, for simplicity they refer to their tests as ‘seasonal unit-root tests’

4 The Canova-Hansen tests

Following the KPSS methodology referred to in the last section Canova and Hansen generalize that test for the seasonal frequencies thus providing a useful complement to the HEGY testing methodology. Taking into account the power properties of the HEGY test these authors propose a “*set of tests to examine the structural stability of seasonal patterns over time. The tests are built on the null hypothesis of unchanged seasonality and can be tailored to test for unit roots at seasonal frequencies or for time variation in seasonal dummy variables.*” (Canova and Hansen, 1995, page 251)

The starting point for these authors is a linear time series model with stationary seasonality which can be specified in two different although mathematically equivalent ways: the first one is the trigonometric representation commonly used in the time series literature;⁷ the second is the dummy formulation. The former gives rise to $s/2$ unit root tests at seasonal frequencies and the latter results in s tests for time variation in the coefficients of the seasonal dummy variables (s being the number of observations per year).

Based on well-established results Canova and Hansen define an LM statistic (called L) which is a function of OLS residuals in a linear model of stationary seasonality, and show that a good test for the null hypothesis of seasonal stationarity against the alternative of seasonal non-stationarity takes the following form: reject H_0 if L is significantly large (a one-tailed test). Being derived from the LM principle the L test statistic is precisely an LM test only if the errors are IID Gaussian variables. Since this is not always a reasonable assumption when dealing with economic data, its relaxation requires modification of the test statistic (by using robust estimates of the variance-covariance matrix) which allows to

7 In this formulation a periodic sequence is represented by a Fourier series, the parameterization of the model uses Fourier coefficients, and seasonality is interpreted as a *cyclical* phenomenon. See Priestley (1981); Aguirre (1995).

interpret it as an ‘LM-like’ test asymptotically equivalent to the true LM test. The authors show that the modified test can be applied to data generated by heteroscedastic and serially correlated processes, among others. Finally, it is shown that the L statistic converges in distribution to a generalized Von Mises distribution with which critical values are obtained.

Depending on the alternative hypothesis of interest different tests result. If the alternative under consideration is ‘seasonal non-stationarity’ then the existence of unit roots at *all* seasonal frequencies should *simultaneously* be tested. This is accomplished by selecting a particular form for the \mathbf{A} matrix appearing in the definition of L , and in that case we have the statistic L_f (subscript ‘f’ indicates that the test is for non-stationarity at *all* seasonal frequencies). If the interest is in testing for seasonal components at specific individual seasonal frequencies the relevant matrix assumes a different form and the original L statistic reduces to $L\theta_j$ ($j = 1, \dots, s/2$) which can be computed as a by-product of the calculation of L_f . When quarterly data are used ($s = 4$) two such statistics result. In the case of monthly data ($s = 12$) this set of statistics has six elements. “*The $L\theta_j$ tests are useful complements to the joint test L_f . If the joint test rejects, it could be due to unit roots at any of the seasonal frequencies. The $L\theta_j$ tests are specifically designed to detect at which specific seasonal frequency non-stationarity emerges.*” (Canova and Hansen, 1995, page 242)

When testing for nonconstant seasonal patterns the more traditional model with seasonal dummy variables is used to determine if the seasonal intercepts change over time. Again, by properly choosing the form of the relevant matrix it is possible to define s different statistics L_a ($a = 1, \dots, s$) which allow testing the stability of the a th seasonal intercept. “*Hence the statistics L_a are essentially the KPSS statistic applied to the seasonal subseries (only the observations from the a th season are used). Then, the KPSS test is for instability in the average level of the series, but the L_a tests are for instability in the seasonal subseries.*” (Canova and Hansen, 1995, page 243) When the objective of the test is the joint stability of the seasonal intercepts an L_f statistic is defined. However, this is a test for instability in *any* of the seasonal intercepts, in such a way that even zero-frequency movements in the series may be detected. As a result, the null hypothesis can be rejected as a consequence of the existence of long-run instability at that frequency, which is an undesirable feature of the test.⁸ The modifications proposed by Canova and Hansen to cope with this problem led them back to the joint test statistic L_f defined in the first case. This

⁸ The authors recognize that this objection is also applicable to the case of the individual test statistics L_a , but the problem is far more acute with the joint test L_f .

result prompts the following declaration from the authors: *“To put the finding in another way, we have found that either construction - testing for instability as viewed through the lens of seasonal intercepts or from the angle of seasonal unit roots gives exactly the same joint test.”*(Canova and Hansen, 1995, page 243)⁹

The large-sample distribution properties of all proposed statistics are established. All asymptotic distributions are of the Von Mises type with different values for the degrees of freedom parameter. As a consequence, the critical values in the table provided in the paper vary with the significance level and the number of degrees of freedom. A Monte Carlo experiment that combines two different models with data obtained from three DGPs studies the comparative performance of the proposed test statistics with two alternative testing methodologies. These results are not very different from those obtained by Franses (1994), Ghysels *et alii* (1994) and Hylleberg (1995), all of them for quarterly data, which show mixed results. Sansó *et alii* (1998), working with monthly data, conduct a large Monte Carlo simulation experiment to study the performance of small sample parametric tests for seasonal stationarity and obtain similar outcomes that can be summarized with this citation: *“In view of these results, we agree with the advise of Canova and Hansen (1995) and Hylleberg (1995) in the sense that it is very convenient to simultaneously use the tests with null of seasonal nonstationarity together with the CH tests. If there is agreement in the evidence obtained from both types of tests, then this can be interpreted as strong evidence. On the contrary, if those methodologies produce different results, then detailed analyses are needed because it is evident that the data do not allow to properly discriminate between the trend-stationary hypothesis and the difference-stationary case.”*(Sansó *et alii*, 1998, our translation)

5 Summary and conclusions

In the same way that a time series (at the zero frequency) may be well described by a deterministic process, a stationary stochastic process or an integrated process, the seasonal components of a time series may be well described by a process of any of those three kinds or by a combination of elements of each. This fact calls for simple statistical techniques that can discriminate between the different possibilities. Depending upon the particular case, seasonal averaging or seasonal differencing can be necessary in order to

⁹ All the CH statistics can be computed with relatively simple routines which can be programmed in such software packages as RATS, SAS (IML procedure), S-PLUS or MATLAB.

achieve stationarity. If the process is integrated, the seasonal component drifts substantially over time. This possibility is implicit in the common practice of taking seasonal differences like suggested by Box and Jenkins (1976).

While it is usual to work with tests that have a null hypothesis of non-stationarity, for some DGPs those tests have low power. Trying to solve this problem other tests were proposed. They use as null the nonexistence of unit roots at seasonal frequencies while the alternative states that there is a unit root at either a single seasonal frequency or a set of them.

It could be argued that, since the consequences of non-stationarity are so important, it is advisable to take a conservative approach and work with non-stationarity as the maintained hypothesis. However, no one will deny that it would be useful to test using both forms of the null, to ensure that each corroborates the other. This is precisely the contribution made by the tests discussed in this note.

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