# Testing for seasonal unit roots in Brazilian monetary series*

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## RESUMO

O objetivo deste artigo é estudar as propriedades de várias séries temporais monetárias brasileiras. Em particular, são aplicados os procedimentos de teste propostos por Beaulieu e Miron (1993) para determinar a presença de raízes unitárias na freqüência zero e/ou nas freqüências sazonais. Estes testes mostram, em todos os casos, a existência de uma raiz unitária na freqüência zero e indicam que não existem raízes unitárias nas freqüências sazonais. Estos resultados implicam que a primeira diferença da cada série é estacionária e pode ser modelada com variáveis *dummy* sazonais.

**Palavras-chave**: variação sasonal, sasonalidade determinística, sasonalidade estocástica.

## ABSTRACT

The objective of this paper is to study the time series properties of several Brazilian monthly monetary series. The test procedures proposed by Beaulieu and Miron (1993) are applied to determine the presence of unit roots at the zero frequency as well as the seasonal frequencies. In all cases these tests point out the existence of a unit root at the zero frequency but do not find any at the seasonal frequencies. These findings imply that the first differences of the series are stationary and can be modelled with seasonal dummy variables.

**Key words**: seasonal variation, deterministic seasonality, stochastic seasonality.

**JEL classification**: C4

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## 1 Introduction

Seasonality is an important feature of many economic time series and needs to be understood and taken into account in the modelling of time series data. It can either be filtered out of the data or jointly modelled with the other characteristics of the phenomenon under study. The first approach involves using a filter to obtain seasonally adjusted data (such an approach was followed by Haache (1974) in studying the demand for money). The second attempts to capture seasonality by means of seasonal dummies, which is equivalent to assuming seasonal variations to be purely deterministic. "*However, if seasonal effects change gradually over time, this (second) approach leads to dynamic misspecification ...*" (Harvey and Scott, 1994, p. 1324) For this reason, whenever seasonal data is used in econometrics, it seems advisable to test for the time series properties of the variables, rather than to assume the appropriateness of any model specification.

The seasonal movement in many economic variables is the result of a complex decision-making process based on varying and changing exogenous causes. Hylleberg (1992, p. 4) proposed the following definition:

> *Seasonality is the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy.*

The last two decades have witnessed a renewed interest in the problem of seasonality. Econometricians have come to agree that seasonal variation accounts for a major part of the total variation in many quarterly and monthly time series. However, agreement is not so strong on the issue of whether the seasonal components are very regular and constant over long periods of time, or whether they change over the years. Another controversy has concerned the nature of the interdependence between the seasonal components and other components of the time series such as the business cycle component.

> *Recent empirical studies suggest that a straightforward incorporation of seasonal fluctuations in econometric models using simple deterministic terms does not seem feasible. This conjecture is based on the following two stylised facts (or empirical regularities). The first is that seasonal fluctuations in many quarterly and monthly observed macroeconomic time series do not appear to be constant over time. The second*

> *is that for several macroeconomic series it appears that the seasonal fluctuations and nonseasonal fluctuations are not independent, in the sense that one may observe different seasonal fluctuations in business cycle expansion periods from those in recession periods.* (Franses, 1996, p. 1)

The immediate implication of the above second regularity is that the key assumption of seasonal adjustment methods - that is, that one can identify independent seasonal and nonseasonal components - does not hold.

The model selection techniques popularised by Box and Jenkins (1976) recommend the use of the seasonal filters $\Delta\Delta_s = (1-L)(1-L^s)$ or simply $\Delta_s = (1-L^s)$ to get rid of seasonal variations in the data.[1] Such filters are appropriate only where the series dealt with is seasonally integrated (the former implies the existence of 13 unit roots and the latter of 12 such roots). However, if fewer unit roots are present, the use of these filters yields an overdifferenced series. For example, where there is only one unit root,[2] applying the $\Delta = (1-L)$ filter would be sufficient to make the series trend stationary, while deterministic seasonality could be handled by the inclusion of seasonal dummies. This overdifferencing may cause problems in the construction of time series models because the (partial) autocorrelation pattern becomes hard to interpret. Furthermore, estimation problems may occur because of the introduction of moving average polynomials with roots close to the unit circle. On the other hand, underdifferenced series may yield unit roots in their autoregressive parts, and classical arguments such as those provided by Granger and Newbold (1974), for time series containing neglected unit roots may apply. So, again, it seems to be important to test for (seasonal) unit roots.

The objective of this paper is to attempt to make a contribution by discussing the application of a testing procedure to determine the seasonal properties of a set of monthly Brazilian monetary series. This kind of empirical study necessarily precedes any further analysis of the seasonality of any monthly series and any cointegration analysis where such series are included.

In Section 2 we present three typical seasonal models used in empirical work. Section 3 provides a brief discussion of unit root tests including the so-called zero frequency unit root

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1 $L$ is the usual lag operator.

2 In some cases there is a single seasonal unit root, requiring the use of a specific filter, e.g. in Abraham and Box's analysis of the famous 'airline passengers' series (1978).

tests, the seasonal unit root tests for quarterly data, and the corresponding tests for monthly data that are used in this paper. The description of the Brazilian monetary series used in this paper and the results of the tests we performed, appear in Section 4, while the last section presents a summary of the paper and our conclusions.

## 2 Modelling seasonal processes

Several different time series models of seasonality can be applied. The most common factors influencing selection are whether seasonal patterns can be represented by deterministic dummies or whether the series is seasonally integrated.[3] These factors are related to the following three classes of processes: purely deterministic seasonal processes, (covariance) stationary processes, and integrated seasonal processes.

The first class includes those processes generated by purely deterministic components such as a constant term and seasonal dummy variables. In the following (simple) example, variable $y_t$ - observed $s$ times each year - is generated solely by seasonal intercept dummies:[4]

$$y_t = \sum_{i=1}^{s} \alpha_i Di_t + \varepsilon_t \tag{1}$$

where the dummy variables $Di_t$ ($i = 1, 2,..., s$) take value 1 when $t$ lies on season $i$, and zero otherwise, and $\varepsilon_t$ is a series of IID random variables. This equation can be reformulated so as to avoid confounding the levels and the seasonals, in the following way:

$$y_t = \mu + \sum_{i=1}^{s-1} \alpha_i^* Di_t^* + \varepsilon_t \tag{2}$$

where $\mu$ is the mean of the process and the coefficients $\alpha_i^*$ are constrained to sum zero. In order to make this constraint operative, the $Di_t^*$ dummies are defined as 1 when $t$ lies in season $i$, $-1$ when $t$ lies in season $s$ and zero otherwise. Finally, the above equation may also include deterministic trends with constant or variable coefficients across seasons, *i.e.*

3 The concept of 'seasonal integration' may mean different things to different authors.

4 A more general model may include an autoregressive and/or a moving average component.

$$y_t = \mu + \sum_{i=1}^{s-1} \alpha_i Di_t + \sum_{i=1}^{s} \beta_i [Di_t \times g(t)] + \varepsilon_t \qquad (3)$$

where $g(t)$ is a deterministic polynomial in $t$.[5]

The second class - covariance stationary seasonal processes - can be exemplified by the model expressed as

$$y_t = \rho y_{t-s} + \varepsilon_t \qquad (4)$$

where $|\rho| < 1$

If $\rho = 1$ in equation (4), we have a *seasonal random walk*, a process that exhibits a seasonal pattern which varies over time. This is the third class of seasonal processes referred to above. In that case, $\Delta_s y_t$, defined as:

$$\Delta_s y_t = y_t - y_{t-s} = \varepsilon_t \qquad (5)$$

is stationary.

The main difference between these forms of seasonality is that in the deterministic and the stationary stochastic seasonal models, shocks die out in the long run while they have a permanent effect in the integrated model. That is to say, seasonally integrated processes have properties similar to those observed in the ordinary (zero frequency) integrated series. As Hylleberg *et al.* have suggested, "*they have 'long memory' so that shocks last forever and may in fact permanently change the seasonal patterns. They have variances which increase linearly since the start of the series and are asymptotically uncorrelated with processes with other frequency unit roots.*" (1990, p. 218)

Three different definitions of seasonal integration are proposed by Osborn *et al.* (1988), Engle *et al.* (1989) and Hylleberg *et al.* (1990). According to Osborn's definition, a variable is said to be integrated of order $(d,D)$ - denoted $I(d,D)$ - if the series becomes stationary after first-differencing $d$ times and seasonal differencing $D$ times. That is to say, $X_t \sim I(d,D)$ if

5 Note that all the above deterministic processes can be forecast and will never change their shape.

$(1-L)^d(1-L^s)^D X_t = \Delta^d \Delta_s^D X_t$ is stationary. Engle's definition states that a time series is integrated of order $d_0$ and $d_s$, denoted $SI(d_0,d_s)$, if $(1-L)^{d_0}[S(L)]^{d_s} X_t = \Delta^{d_0}[S(L)]^{d_s} X_t$ is stationary, where the polynomial expression *S(L)* is defined as $S(L) = 1 + L + L^2 + \ldots + L^{S-1}$ [6]

When variables do not present seasonal integration, both definitions coincide, *i.e.*, I(1,0) = *SI*(1,0), *I*(2,0) = *SI*(2,0), etc. On the other hand, whenever a series is seasonally integrated these definitions differ. This is so because $\Delta_s = (1 - L^s)$ can be factored into $(1 - L)S(L)$. In this way, the equivalent of *I*(0,1) is *SI*(1,1); *I*(1,1) = SI(2,1), and so on. In the same way, the *SI*(0,1) process - using Engle's definition - does not have an equivalent if we use Osborn's concept.

Finally, Hylleberg's definition states that "*a series $x_t$ is an integrated seasonal process if it has a seasonal unit root in its autoregressive representation. More generally it is integrated of order d at frequency $\theta$ if the spectrum of $x_t$ takes the form*

$$f(\omega) = c(\omega - \theta)^{-2d}$$

for $\omega$ near $\theta$. This is conveniently denoted by $x_t \sim I_\theta(d)$." (Hylleberg *et al.* 1990, p. 217)

Engle's *SI* definition will be the one used in this paper, because that is the definition used by Beaulieu and Miron (1993) (B&M from now on) whose methodology we used in our study.

The strict interpretation of seasonal integration in any data-generating process implies that 'summer may become winter' in the sense that the seasonal pattern may change dramatically. For this reason, the finding of one or more seasonal unit roots may indicate a varying and changing seasonal pattern, *i.e.*, evidence against a constant seasonal pattern. As Hylleberg pointed out:

> *whether the seasonal unit root is the result of variation and changes in seasonal causes like the weather, or seasonal mean shifts due to interdependencies between the business cycle and the seasonal pattern ..., or of other changes is a question which requires a much deeper analysis than a univariate test can provide.* (1992, p. 11)

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This polynomial is related to the decomposition of the $(1-L^s) = 0$ polynomial.

The interpretation of seasonal cointegration - as defined in Hylleberg *et al.* (1990) - follows a similar line of reasoning and can indicate a parallel but varying seasonal component in a set of time series.

For all the above reasons, the statistical techniques designed to test for seasonal integration deal with tests to check for the presence of unit roots.

## 3 Unit root tests

The detection of unit roots was first studied in relation to annual data (the so-called zero frequency). The extension of the resulting methodologies to include seasonal frequencies occurred in two stages - first in relation to quarterly data (zero plus three seasonal frequencies) - and later in relation to monthly data (zero plus eleven seasonal frequencies). As soon as the new methods became available, alternative procedures were proposed. In this way, not only parametric tests but also semiparametric, nonparametric and Bayesian techniques were developed. For each of them the three-stage process was a natural development. Furthermore, in each case different possibilities were suggested concerning the form of the null and alternative hypotheses, not to mention a large number of different data generating processes. The consideration of broken trend alternative hypotheses added even more material to this huge body of research.

The history of (non-seasonal) unit root tests started with Dickey and Fuller (1979) and the well-known Augmented Dickey-Fuller (ADF) test with a non-stationary model as the null hypothesis.

The first test for seasonal integration resembled a generalisation of the ADF test for integration in annual data. Dickey, Hasza and Fuller (1984) (DHF from now on), following the methodology suggested by Dickey and Fuller (1979) for the zero-frequency unit-root case, proposed a test of the hypothesis $\rho = 1$ against the alternative $\rho < 1$ in the model $y_t = \rho y_{t-s} + \varepsilon_t$. The DHF test - as well as similar ones proposed in the following years - only allows for unit roots at all of the seasonal frequencies and has an alternative hypothesis which is considered rather restrictive, namely that all the roots have the same modulus. Trying to overcome these drawbacks Hylleberg *et al.* (1990) (from now on referred to as HEGY) proposed a more general testing strategy that allows for unit roots at some (or even all) of the seasonal frequencies as well as the zero frequency. HEGY's methodology allows testing for the existence of unit roots at some seasonal frequencies without arguing in favour of the presence of these kinds of roots at all seasonal frequencies.

### 3.1 Seasonal unit root tests with monthly data

The HEGY (1990) procedure for quarterly data was extended in relation to monthly data in two different - though similar - directions. Franses (1991a, 1991b) discussed a method to distinguish empirically between models (2) and (5) presented above.[7] In his second paper this author showed that conventional autocorrelation checks cannot generally make this distinction because they are not discriminative. He also showed that considering a model like (5), or similar, when (2) is more appropriate yields a deterioration of forecasting performance.

B&M (1993) used - in a slightly different way - the approach developed by HEGY to derive the mechanics of another procedure to test for seasonal unit roots using monthly data. These authors derived the asymptotics of HEGY's procedure for monthly data, and also computed the finite sample critical values of the associated test statistics using Monte Carlo methods. The main difference compared with Franses' (1991a, 1991b) methodology is that B&M used mutually orthogonal regressors, obtaining a different - somewhat more complicated - test equation.

Suppose that the series of interest ($X_t$) is generated by a general process like:

$$\varphi(L)\,X_t = \alpha_0 + \alpha_1 t + \sum_{k=2}^{12} \alpha_k D_{kt} + \varepsilon_t \qquad (6)$$

where $\varepsilon_t$ is a white noise process and the deterministic terms include a constant, a linear trend and seasonal dummies. The question examined by B&M was:

> *whether the polynomial in the backshift operator, $\varphi(L)$, has roots equal to one in absolute value at the zero or seasonal frequencies. In particular, the goal is to test hypotheses about a particular unit root without taking a stand on whether other seasonal or zero frequency unit roots are present.* (1993, p. 307)

The auxiliary regression model that allows the test to be performed is provided by the following equation:

7 Actually, Franses' models are more general, since they include autoregressive and moving average parts, but the distinction between deterministic and stochastic seasonality is the same.

$$\varphi(L)^* Y13_t = \alpha_0 + \alpha_1 t + \sum_{k=2}^{12} \alpha_k D_{kt} + \sum_{k=1}^{12} \pi_k Yk_{t-1} + \varepsilon_t \qquad (7)$$

where $Yk_t$ ($k$ = 1, 2, ..., 12) are auxiliary variables obtained by appropriately filtering the variable ($X_t$) under study.[8] The $\varphi(L)^*$ polynomial is a remainder with roots outside the unit circle which allows the augmentation necessary to whiten the errors in the estimation of the above equation. "*In order to test hypotheses about various unit roots, one estimates [the test equation] by Ordinary Least Squares and then compares the OLS statistics to the appropriate finite sample distributions based on Monte Carlo results.*" (B&M, 1993, p. 309) The inclusion or not of a trend in the deterministic part of model (7) depends upon the hypothesised alternative to the null hypothesis of 12 unit roots.

So, there are twelve possible unit roots, one non-seasonal and eleven seasonal. Out of the eleven seasonal unit roots one is real and the other ten form five pairs of complex conjugates.[9] B&M provide the asymptotic distribution of the statistics necessary to perform the tests: $t_1$, $t_2$, $t_k$ and $t_{k+1}$, where $k \in \{3, 5, 7, 9, 11\}$ They also prove that the asymptotic distribution of the five $t_k$ statistics are the same as those of the five $t_{k+1}$.

For ease of notation B&M considered that $k$ is 'odd' if $k \neq 1$ with $k \in \{3, 5, 7, 9, 11\}$ and that is 'even' if $k \neq 2$ with $k \in \{4, 6, 8, 10, 12\}$. These authors studied the distributions of the $t$-statistics under different conditions; first, when no deterministic terms are included, and second when different combinations of constant, seasonal dummy and trend terms are included in the regression. Their theoretical results showed "*that all of the odd statistics have the same distributions when different deterministic regressors are included in the regression. The same is true for the even statistics. One can also see that $t_1$ is invariant to the inclusion of seasonal dummies as long as a constant is included.*" (B&M, 1993, p. 315) They also showed that "*because all odd t-statistics have the same distribution and all even t-statistics have the same distribution, all of the F-statistics have the same distribution for any set of included deterministic regressors.*" (B&M, 1993, p. 316) The distributions of $t_2$, ..., $t_{12}$ are independent of a constant or a constant plus trend terms. The inclusion of any of these

8 The definitions of these auxiliary variables are reproduced in Appendix I. For details see Hylleberg *et al.* (1990) and B&M (1993).

9 See Appendix I.

alternative sets of terms only modify the distribution of $t_1$. The finite sample distributions obtained by Monte Carlo methods display all the characteristics of the theoretical asymptotic distributions.

The test of the null hypothesis that there is a unit root in a given frequency is carried out by testing the significance of the corresponding $\pi$ coefficient, estimated with equation (7). The appropriate critical values for such tests are those provided by the finite sample distributions based on Monte Carlo results.

If all the estimated coefficients are statistically different from zero, the series present a stationary seasonal pattern and no further filtering is necessary. In case $\pi_i = 0$, for $i = 1, ..., 12$, the series is seasonally integrated and it is appropriate to use the seasonal difference filter $(1 - L^{12})$.

If only $\pi_1 = 0$, then the presence of a root equal to +1 at the zero frequency cannot be rejected. There will be no seasonal unit roots if $\pi_2$ through $\pi_{12}$ are significantly different from zero. When only some pairs of $\pi$'s are equal to zero, one should consider using the corresponding implied operators. Abraham and Box (1978) showed how these kinds of operators may sometimes be enough.

To be more specific, the null about the presence of a unit root at the zero frequency is tested with the "$t$" statistic of the hypothesis $H_0$. $\pi_1 = 0$ (called $t_1$ by B&M). The null hypotheses about the existence of seasonal unit roots are tested, in each frequency, by means of the corresponding "$t$" statistic associated with $H_0$. $\pi_i = 0$, for $i = 2, 3, ..., 12$, and/or by means of the "$F$" statistics corresponding to the joint hypotheses $H_0$. $\pi_i = \pi_{i+1}$, for $i = \{3, 5, 7\ 9, 11\}$ which take into account all pairs of conjugate complex roots.[10] The significance tests for $\pi_1$ and $\pi_2$ are one-sided as well as those corresponding to $\pi_i$ for 'even' $i$. On the other hand, those corresponding to 'odd' values of $i$ should be two-sided.

## 4 Brazilian monetary series

In this section we report on our testing of several Brazilian monetary series for the presence

10 In the case of pairs of complex roots we reject the null hypothesis if any one of the '$t$' statistics is significant.

of seasonal unit roots. The series were M1, M2, M3 and M4, Currency (issued currency), Reserves (bank reserves deposited in the Central Bank) and the Monetary Base. The source of the original data is the Internet site of the Brazilian Central Bank.[11] The nominal values were expressed in 'reais' with constant purchasing power by means of the IGP/DI price index estimated by the Fundação Getúlio Vargas.

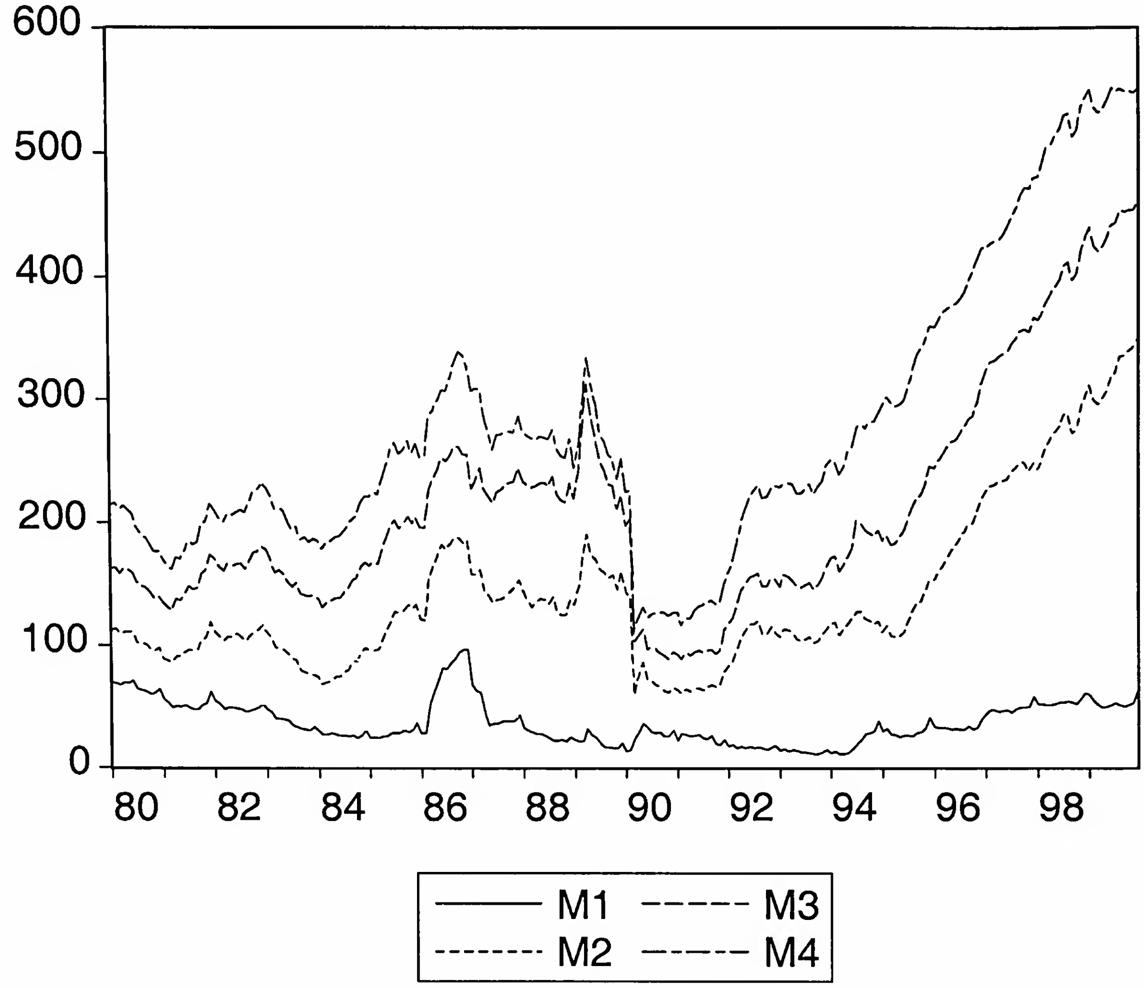
The definitions of these series are as follows. The end of month narrow money stock (M1) is formed by the non-financial private sector holdings of currency plus demand deposits. M2 is equal to M1 plus Mutual Funds ('Fundos de Aplicações Financeiras') plus Money Market Accounts ('Fundos de Investimentos Financeiros-Curto Prazo'). Adding saving deposits to M2, we obtain M3. Finally, M4 is equal to M3 plus time deposits. The Monetary Base is formed by issued currency plus bank reserves deposited in the Central Bank.

All these series, presented in Figures I and II, show a seasonal high level in December. This seasonality is explained mainly by the payment of a Christmas bonus (usually known as 'the thirteenth salary') to all employees across the country, and by the high consumption expenses related to the Christmas season.

Starting in February 1986 several stabilisation plans were implemented in Brazil with the objective of fighting inflation. Most of these plans included a price and wage freeze and the elimination of automatic indexing based on past inflation. Because of their very nature, all these plans reduced the opportunity cost of holding currency and demand deposits. Consequently, in the months following each one of these plans, there occurred a fast and intense substitution of currency and demand deposits for non-monetary financial assets. In Figure I it is easy to observe the monetisation of the economy in 1986 after the so-called 'Plano Cruzado I' The drastic increase in M1 was also short-lived, since the 1986 increment disappeared in 1987 Starting in 1994, when the 'Plano Real' was implemented, the issuing of currency was severely tightened. Despite this control, M1 grew steadily after that year as the result of the monetisation of the economy. However, as the issuing of Treasury Bills and similar bonds grew considerably more in the same period, from that date on it is observed a strikingly fast increase in the M2, M3 and M4 series.

11 The interested reader may obtain these series from the following e-mail address: aguirre@cepe.ecn.br.

**Figure I**
**Brazil - End of Period Money Supply in Real Terms**
**Jan/1980-Dec/1999**

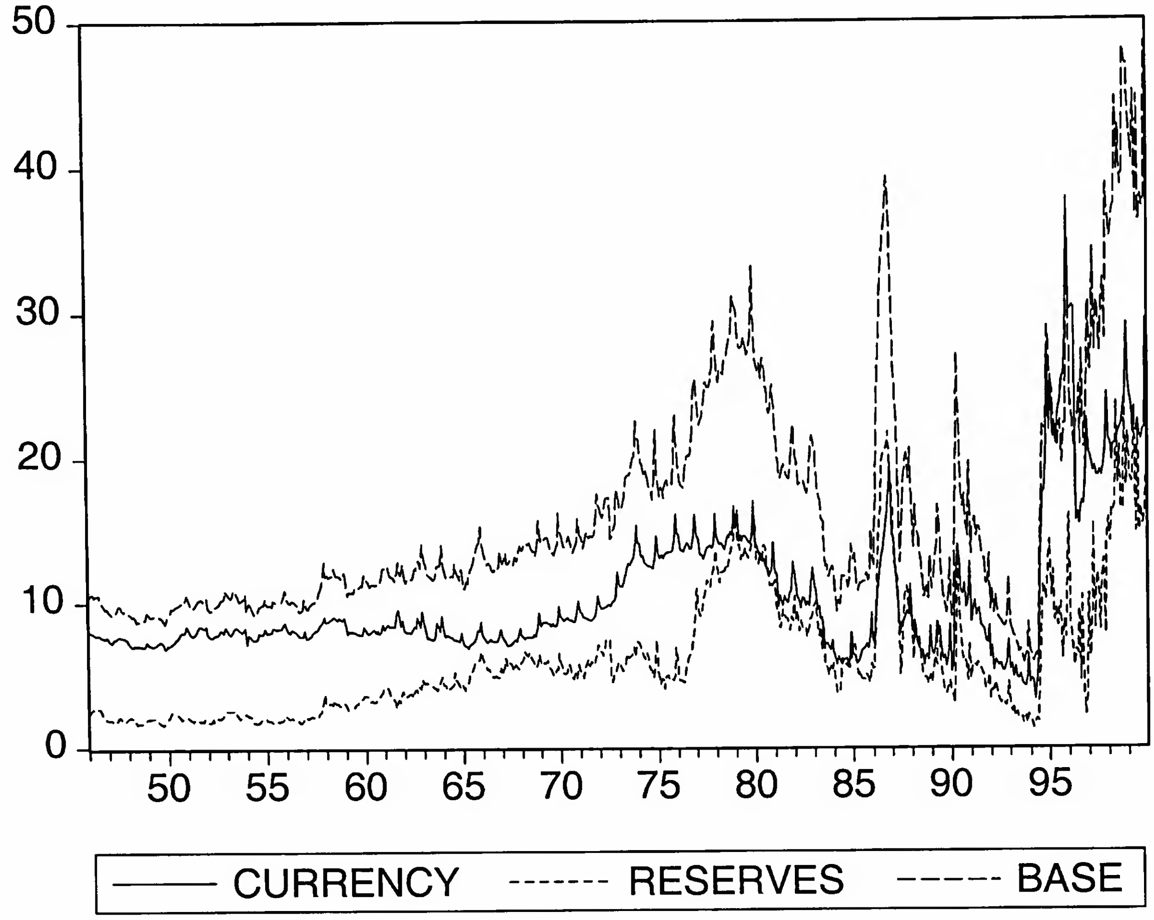



Source: see text.

When the first stabilisation plans were implemented, the existence of a significant volume of demand deposits and currency increased the demand for real assets, stocks and foreign exchange in such a way that the situation was not compatible with price stability. For this reason, when the Collor Plan was implemented in 1990 it was decided that, in addition to altering the monetary standard, a substantial share of the financial assets in the economy would be made temporarily non-convertible. The government froze this large share of all financial assets as a means of gaining control over the inflationary process. In other words, all bank accounts were temporarily seized up. The initial volume of resources seized by the Central Bank came to approximately two thirds of the money supply in its broadest (M4) concept.[12] This fact explains the violent decrease of the M2, M3 and M4 series in 1990 (see Figure I).

12 See Bacen, 1990.

**Figure II**
**Brazil - End of Period Reserves, Issued Currency and Monetary Base in Real Terms**
**Jan/1946-Dec/1999**




Source: see text.

For all these series we applied OLS to the auxiliary regression (7) in order to obtain the estimates of $\pi_i$ and the corresponding standard errors.[13] The *t*-statistics calculated with these estimates are compared with the critical values from the small sample distributions based on Monte Carlo studies and published by B&M (1993) to perform the statistical tests. Table I presents a summary of the results obtained after performing the B&M tests in order to check for the integration of the series in its seasonal and nonseasonal parts, under the null hypotheses that the series are $SI(1,1)$.

13 The truncation lag was chosen using the general to specific recursive method with kmax = 12.

**Table I**
**Regression Results to Test for Unit Roots**

| Null Hypotheses | M1 | M2 | M3 | M4 | Currency | Base | Reserves |
|---|---|---|---|---|---|---|---|
| $\pi_1 = 0$ | -2.29 | 0.21 | - 0.32 | 0.37 | 0.68 | 0.32 | -2.00 |
| $\pi_2 = 0$ | -6.60* | -3.39* | -3.37* | -3.33* | -3.86* | -8.55* | -7.29* |
| $\pi_3 = 0$ | -3.88* | -3.55* | -3.52* | -2.60 | -5.38* | -8.12* | -7.50* |
| $\pi_4 = 0$ | -5.05* | -5.28* | -5.63* | -5.73* | -3.93* | -6.53* | -5.11* |
| $\pi_5 = 0$ | -5.75* | -5.96* | -5.93* | -5.52* | -5.57* | -10.7* | -8.25* |
| $\pi_6 = 0$ | 2.38* | 3.59* | 3.23* | 2.83* | 5.59* | 1.03 | -3.81* |
| $\pi_7 = 0$ | -1.55 | -2.33 | -2.21 | -2.18 | -5.59* | -5.29* | -5.55* |
| $\pi_8 = 0$ | -5.55* | -6.58* | -6.76* | -6.80* | -5.75* | -6.75* | -5.99* |
| $\pi_9 = 0$ | -8.38* | -5.56* | -5.70* | -5.97* | -5.78* | -11.92* | -11.75* |
| $\pi_{10} = 0$ | 1.30 | 2.37* | 2.17* | 1.65 | -1.91* | 0.55 | 0.38 |
| $\pi_{11} = 0$ | 2.53 | -1.00 | -1.35 | -1.56 | 1.99 | 0.37 | - 0.86 |
| $\pi_{12} = 0$ | -5.27* | -7.01* | -6.90* | -7.13* | -6.72* | -10.92* | -9.30* |
| $\pi_3 = \pi_4 = 0$ | 17.05* | 22.17* | 25.26* | 20.99* | 22.10* | 59.09* | 53.35* |
| $\pi_5 = \pi_6 = 0$ | 15.80* | 20.05* | 18.73* | 19.97* | 20.76* | 57.58* | 52.52* |
| $\pi_7 = \pi_8 = 0$ | 11.79* | 25.73* | 26.66* | 26.82* | 27.52* | 33.65* | 27.18* |
| $\pi_9 = \pi_{10} = 0$ | 36.60* | 19.21* | 19.58* | 19.66* | 19.25* | 71.59* | 69.08* |
| $\pi_{11} = \pi_{12} = 0$ | 18.03* | 25.50* | 25.18* | 27.07* | 25.59* | 59.69* | 53.36* |
| Lagged terms | 0 | 0 | 0 | 0 | 2 | 0 | 1 |

(*) Significant at 5% level. Critical values given by B&M (1993).
The test regressions have a constant and eleven dummies.
Source: see text.

In the case of all seven series, the data rejected the presence of unit roots at all seasonal frequencies. However, the existence of a unit root at the zero frequency could not be rejected. These results imply that the seasonality present in these monthly series is partly deterministic and partly stationary stochastic and, as a consequence, no seasonal differencing is necessary in order to obtain stationarity. The presence of a nonseasonal unit root in each series, however, requires the use of first differences.

## 5 Summary and conclusions

Non-adjusted quarterly (or monthly) economic time series showing seasonal patterns shed some doubts on the assumption of stationary first differences. The question of whether these seasonal patterns should be eliminated by regression on seasonal dummies (the 'deterministic' model) or by treating them with seasonal differences, thereby assuming the existence of additional unit roots on the unit circle (the 'stochastic' model), resembles the old discussion of deterministic and stochastic trend models. The existence of unit roots at the seasonal frequencies has similar implications for the persistence of shocks as in the case of the existence of a unit root at the zero frequency.

The standard Box-Jenkins (1976) approach popularised the use of estimated autocorrelation functions to identify a tentative time series model. This method implies that the double differencing filter $\Delta\Delta_s = (1-L)(1-L^s)$ is useful to remove unit roots from a seasonal time series. This usually applied double differencing filter may be superfluous in some circumstances. If the double filter, or simply the $\Delta_s$ filter is required, the time series is said to be seasonally integrated.

Since differencing filters assume the presence of one or more seasonal or non seasonal unit roots, most methods to test for an appropriate differencing filter are based on statistical tests for the presence of such unit roots. These tests are all extensions of the well-known Dickey-Fuller (1979, 1981) tests. B&M (1993) present a reformulation of an autoregression, isolating the key unit root parameters in the case of monthly data.[14] Based on least-squares fits of univariate autoregressions on transformed variables similar to the augmented Dickey-Fuller regression, B&M developed tests for the existence of seasonal as well as zero-frequency unit roots in monthly data and tables of the corresponding critical values.

In our analysis we used formal testing procedures that were put forward in the last decade to investigate the adequacy of the use of seasonal filters in the case of some Brazilian monthly monetary series. In particular, we used the methodology proposed by B&M (1993) to test for the presence of unit roots, be they at the zero or seasonal frequencies. We did not find any evidence in favour of seasonal integration, since we rejected unit roots at all the seasonal frequencies in all the series we considered. However, the presence of a unit root at the zero frequency could not be rejected in any case. All these results imply that the univariate representation of these monetary time series is a difference stationary process around a deterministic seasonal pattern represented by seasonal intercept dummies.

14 They do so by expanding the procedures presented by Hylleberg *et al.* (1990) for quarterly data.

## Appendix I - Seasonal unit roots

If a series has a seasonal pattern, then the differencing which removes seasonality should be of degree $s$ rather than one, *i.e.* an operator $y_t - y_{t-s}$ should be applied rather than $y_t - y_{t-1}$. Often $s$–differencing also removes a trend - unless the trend is non-linear, in which case it may be necessary to take first differences of the $s$–differences in order to make the series stationary.

In the case of monthly data, the characteristic equation $(1 - L^{12})$ associated with the seasonal differencing operator has twelve roots on the unit circle. Using these unit roots, the polynomial can be written as the product of twelve factors each of them involving one of the roots. Based on this decomposition, and using $X$ to stand for the variable under analysis, the following auxiliary variables were defined and calculated in order to perform the tests reported in section 4:[15]

$$Y1_t = (1 + L + L^2 + L^3 + L^4 + \cdots + L^{11})X_t$$

$$Y2_t = -(1 - L + L^2 - L^3 + \cdots - L^{11})X_t$$

$$Y3_t = -(L - L^3 + L^5 - L^7 + L^9 - L^{11})X_t$$

$$Y4_t = -(1 - L^2 + L^4 - L^6 + L^8 - L^{10})X_t$$

$$Y5_t = -\tfrac{1}{2}(1 + L - 2L^2 + L^3 + L^4 - 2L^5 + L^6 + L^7 - 2L^8 + L^9 + L^{10} - 2L^{11})X_t$$

$$Y6_t = \frac{\sqrt{3}}{2}(1 - L + L^3 - L^4 + L^6 - L^7 + L^9 - L^{10})X_t$$

$$Y7_t = \tfrac{1}{2}(1 - L - 2L^2 - L^3 + L^4 + 2L^5 + L^6 - L^7 - 2L^8 - L^9 + L^{10} + 2L^{11})X_t$$

15 For details see Beaulieu and Miron (1993). Franses' (1991a, 1991b) alternative methodology uses only seven auxiliary variables to perform this test.

$$Y8_t = -\frac{\sqrt{3}}{2}(1 + L - L^3 - L^4 + L^6 + L^7 - L^9 - L^{10})X_t$$

$$Y9_t = -\tfrac{1}{2}(1 - L - 2L^2 - L^3 + L^4 + 2L^5 + L^6 - L^7 - 2L^8 - L^9 + L^{10} + 2L^{11})X_t$$

$$Y9_t = -\tfrac{1}{2}(1 - L - 2L^2 - L^3 + L^4 + 2L^5 + L^6 - L^7 - 2L^8 - L^9 + L^{10} + 2L^{11})X_t$$

$$Y10_t = \tfrac{1}{2}(1 - \sqrt{3}L + 2L^2 - \sqrt{3}L^3 + L^4 - L^6 + \sqrt{3}L^7 - 2L^8 + \sqrt{3}L^9 - L^{10})X_t$$

$$Y11_t = \tfrac{1}{2}(\sqrt{3} + L - L^3 - \sqrt{3}L^4 - 2L^5 - \sqrt{3}L^6 - L^7 + L^9 - \sqrt{3}L^{10} + 2L^{11})X_t$$

$$Y12_t = -\tfrac{1}{2}(1 + \sqrt{3}L + 2L^2 + \sqrt{3}L^3 + L^4 - L^6 - \sqrt{3}L^7 - 2L^8 - \sqrt{3}L^9 - L^{10})X_t$$

The last auxiliary variable, defined as $Y13_t = (1 - L^{12})X_t$, is the dependent variable in equation (7) of section 3.1.

Abraham and Box (1978) factorise the $(1 - L^{12})$ operator in a different way:

$$(1 - L^{12}) = (1 - \sqrt{3}L + L^2)(1 - L + L^2)(1 + L^2)(1 + L + L^2)(1 + \sqrt{3}L + L^2)(1 + L)(1 - L)$$

The corresponding roots, periods and frequencies associated with each one of these factors are given in a table that we reproduce below.

**Table A1**

| | Factor | Root | Period | Frequency in cycles per year |
|---|---|---|---|---|
| 1 | $1-\sqrt{3}L+L^2$ | $(\sqrt{3}\pm i)/2$ | 12 | 1 |
| 2 | $1-L+L^2$ | $(1\pm i\sqrt{3})/2$ | 6 | 2 |
| 3 | $1+L^2$ | $\pm i$ | 4 | 3 |
| 4 | $1+L+L^2$ | $(-1\pm i\sqrt{3})/2$ | 3 | 4 |
| 5 | $1+\sqrt{3}L+L^2$ | $(-\sqrt{3}\pm i)/2$ | 12/5 | 5 |
| 6 | $1+L$ | –1 | 2 | 6 |
| 7 | $1-L$ | 1 | | Constant |

Source: Abraham and Box (1978, p. 130).

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