

The Brazilian experience on estimating consistent interregional systems, the SUIT and IIOAS methodologies compared

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Abstract

In Brazil, as in many other countries, there is no official data on trade flows between subnational regions. Therefore, the estimation of inter-regional input-output systems, under conditions of limited information, has been the subject of researchers at the University of São Paulo Regional and Urban Economics Lab - NEREUS for over two decades. During this span, they have developed two distinctive estimation methods: Supply and Use Interregional Tables (SUIT) and Interregional Input-Output Adjustment Systems (IIOAS). This paper aims to provide a comprehensive overview of these two methods, which were employed to estimate an interregional system encompassing the 27 states within the Brazilian economy. Following the exposition of SUIT and IIOAS methods, a comparative analysis is conducted to delineate their main similarities and differences, in both holistic and partitive terms. The findings underscore that, despite notable differences, particularly in partitive terms, the choice between these two methods generally does not compromise the overall results of structural input-output analysis. Furthermore, this study represents a contribution to the literature, as both SUIT and IIOAS can be applied in the estimation of interregional input-output systems for any country that discloses its Supply and Use Tables and possesses some subnational information for regionalization.

Keywords

Input-output analysis; Estimation of interregional systems; Holistic comparison.

A experiência brasileira na estimação de sistemas inter-regionais consistentes, os métodos SUIT e IIOAS comparados

Resumo

No Brasil, como em muitos outros países, não existem dados oficiais para os fluxos de comércio entre regiões subnacionais. Então, a estimação de sistemas inter-regionais de insumo-produto,

Revisado: 21/12/2023. Aceito: 20/03/2024.

DOI: https://doi.org/10.1590/1980-53575425cgjg



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em condições de informação limitada, tem sido tema de pesquisadores do Núcleo de Economia Regional e Urbana da Universidade de São Paulo – NEREUS, por mais de duas décadas. Nesse período, podem-se destacar a criação de dois métodos de estimação: *Supply and Use Interregional Tables* – SUIT e *Interregional Input-Output Adjustment Systems* – IIOAS. Esse artigo descreve esses dois diferentes métodos, estimando um sistema inter-regional para os 27 estados da economia brasileira. Posteriormente, os sistemas inter-regionais estimados pelo IIOAS e SUIT são comparados, no intuito de identificar suas principais similaridades e diferenças em termos holísticos e partitivos. Os resultados mostram que, apesar das diferenças, principalmente em termos partitivos, em geral a escolha entre esses dois métodos não compromete os resultados da análise estrutural de insumo-produto. Além disso, esse artigo contribui para a literatura, dado que ambos os métodos podem ser aplicados na construção de sistemas inter-regionais de insumo-produto para qualquer país que publique matrizes de Usos e de Produção e possua alguma informação subnacional para regionalização.

Palayras-chave

Análise de insumo-produto; Estimação de sistemas inter-regionais; Comparação holística.

JEL Classification

C67; D57; R15.

1. Introduction

Economic progress is not uniformly distributed across regions; rather, it tends to concentrate around specific focal points. This spatial concentration can lead to substantial economic disparities within a country, as highlighted by Hirschman (1977), who also postulates that economic growth is not confined to the region where it originates but can radiate and impact other regions. Consequently, effective economic policies must consider this regional interdependence, to achieve satisfactory results in terms of overall regional development (Yamano, 2017).

The economic inequality among Brazilian states serves as a clear illustration. For instance, São Paulo, covering less than 3% of Brazil's land area, contributes to 32% of the country's total production. In contrast, Pará, representing 15% of the national territory, contributes just over 2% to the overall production.¹

Considering such disparities and distinct productive structures among Brazilian states, understanding how interstate economic relations form and how the growth of one state affects others becomes crucial. While

According to IBGE, Regional Accounts 2015



Estud. Econ., São Paulo, vol.54(2), e53575425, 2024

the Brazilian Institute of Geography and Statistics (IBGE) regularly publishes national input-output matrices, they treat the entire country as a single region, overlooking the intricate interdependence among states, particularly concerning trade flows. Isard (1998) suggests that interregional input-output systems, unlike national tables, can effectively capture such interdependence and productive spillovers between regions.

Applications of interregional input-output systems span various fields. Among the recent studies we can refer to Duan et al. (2023) on export growth's influence on regional inequality in China, Gonçalves Jr et al. (2022) analyzing illegal trade practices along the Brazilian border and their regional economic impacts, and Brown et al. (2020) examining interprovincial trade and economic integration in Canada.

Consequently, interregional input-output systems serve as powerful tools for policy makers, aiding them in allocating scarce resources to promote regional development (Isard, 1998). Despite significant advancements since Isard (1951) and Leontief et al. (1953), early challenges into regional and interregional extensions of input-output models persist. The scarcity of information, high survey costs, and difficulties in reconciling seemingly incompatible databases, especially regarding interregional trade flows, have hindered the estimation of interregional input-output systems. As a response, non-survey estimation methods have gained academic popularity (Round, 1983; Park et al., 2009).

In special, the development of subnational, regional and interregional, input-output models has garnered global attention among researchers. Some recent works include: (i) Zhang, Shi, and Zhao (2015), who built an interregional input-output system for 30 Chinese provinces; (ii) Tobben and Kronenberg (2015), who updated the CHARM (Cross-hauling adjusted regionalization Method) for more than two regions; and (iii) Haddad et al. (2016), who established an interregional input-output system for 33 Colombian regions, and (iv) Park et al. (2009) which measured internoty spillovers of greenhouse gas emissions associated with economic activity changes in Southern California (US) using a pseudo "top-down" method. All these endeavors were conducted under conditions of limited information.

In Brazil, as in many countries, there is an absence of official data on trade flows between subnational regions. Consequently, researchers at the University of São Paulo Regional and Urban Economics Lab - NEREUS

have grappled with estimating interregional input-output systems under conditions of limited information for over two decades. Notably, they have developed two estimation methods: Supply and Use Interregional Tables (SUIT) and Interregional Input-Output Adjustment Systems (IIOAS).

Several studies have been conducted using these methods. With SUIT, examples include: (i) Guilhoto et al. (2019), who estimated an interregional input-output system for the 27 Brazilian states with 68 sectors and 128 products; (ii) Guilhoto et al. (2010), who constructed an interregional input-output system for the Brazilian Northeast states; (iii) Ichihara and Guilhoto (2008), who estimated an inter-municipal input-output system for São Paulo state municipalities; and (iv) Guilhoto and Sesso Filho (2005b), who constructed an interregional system for the 9 states of Amazônia's Bank and the rest of Brazil for 1999. Using IIOAS, examples include: (i) Haddad, Gonçalves Jr., and Nascimento (2017), who used IIOAS to estimate an interregional input-output system for the 27 Brazilian states with 68 sectors and 128 products; (ii) Porsse, Haddad, and Pontual (2003), who estimated an interregional matrix for Rio Grande do Sul state and the rest of Brazil; and (iii) Domingues and Haddad (2002), who developed an interregional system for Minas Gerais state and the rest of Brazil.

This paper aims to analyze the similarities and consistencies of these two methods by constructing an interregional input-output system for the 27 Brazilian states using each method. The two systems will then be compared in both partitive and holistic terms. Rather than proposing enhancements or criticisms, the focus is on providing detailed descriptions, applying these methods, and comparing results to examine whether the choice of method influences the outcomes of the analysis.

Both SUIT and IIOAS systematically integrate widely applied techniques in constructing interregional input-output systems, such as: (i) locational quotients; (ii) inter-industry locational quotients; and (iii) the iterative RAS procedure. This paper approach enables other researchers to understand and reproduce the process step by step, contributing to the literature on the estimation of interregional systems.

In terms of application, we will demonstrate that IIOAS offers greater ease for systems among regions of the same level, e.g., the 27 Brazilian states. Conversely, SUIT exhibits greater flexibility when combining regions of different levels, such as municipalities and states in the same system.



The primary advantages of SUIT and IIOAS lie in their combination of bottom-up and top-down approaches. These methods ensure consistency with national Supply and Use Tables (SUTs) while preserving the economic peculiarities of each region. Additionally, both methods can be applied in constructing interregional input-output systems for any country that discloses its Supply and Use Tables and possesses some subnational information for regionalization.

2. SUIT and HOAS Methods

In Brazil, as in many other countries worldwide, the challenge of an absence of information for constructing an input-output interstate system is in evidence. To overcome this, data from the Regional Accounts and surveys conducted by the Brazilian Institute of Geography and Statistics (IBGE) are combined with non-survey estimation techniques to estimate interregional input-output systems.

This section, drawing on the work of Guilhoto et al. (2019) and Haddad, Gonçalves Junior, and Nascimento (2017), is dedicated to presenting the SUIT and IIOAS methods. These methodologies blend top-down and bottom-up approaches, wherein subnational information capturing the nuances of each state is integrated with data from the national Supply, Use, Imports, and Taxes tables. This integration ensures the consistency of the estimated systems with the National Accounts. Moreover, both SUIT and IIOAS methods employ hybrid techniques, incorporating both survey and estimated data to construct the interregional system for the 27 states, also referred as being a Unit of the Federation (UF) in this paper.

Both methods were applied to estimate an interregional input-output system for the 27 states, utilizing the national input-output system for 2011, consisting of 68 sectors of activity and 128 products. The national input-output system comprises the: (i) Supply table, (ii) Use table, (ii) Imports table, and (iv) Taxes table, which were estimated through the method presented by Guilhoto and Sesso Filho (2005a) and Guilhoto and Sesso Filho (2010).

The databases utilized for the disaggregation of the national tables include information from the Regional Accounts, official surveys by IBGE, Annual Report of Social Information – RAIS, Department of Federal Revenue of Brazil - RFB, Brazil's National Treasury, among others. These databases

are the source data for the SUIT and IIOAS methods, ensuring that any differences between the two estimated interregional input-output systems, stem solely from the specificities of each method.

2.1. Supply and Uses Interregional Tables - SUIT

Figure 1 illustrates the process of constructing the interregional system using SUIT, a process that will be detailed in this section.

The estimation process starts with the Regional Supply Tables, which provides information on the sectoral production of each state, by each state source industry. The estimation of the Supply Tables for each state involves estimating the States Gross Output for the 68 sectors and 128 products, utilizing data from the Regional Accounts and official surveys published by IBGE. Initially, the Regional Supply Tables maintain the same structure as the national Supply Table. To achieve this, the National Coefficients are constructed according to the Equation:

$$CP_{s,p}^{N} = MP_{s,p}^{N} * inv(diag(e'_{1,s} * MP_{s,p}^{N})) \ \forall s = 1, ..., 68 \ e \ p = 1,...,128;$$
 (1)

Where *inv* means inverse, *diag* means diagonalized vector, s is the sector; p is the product; $MP_{s,p}^N$ is the National Supply Table; $e'_{1,s}$ is a vector of ones. Then, the Regional Gross Output of each sector in each UF are multiplied by $CP_{s,p}^N$ and we have the first estimate of the Regional Production Tables $MP_{s,p}^{UF}$, according to the Equation:

$$MP_{s,p}^{UF} = diag(VBP_s^{UF}) * CP_{s,p}^N \quad \forall s = 1, ..., 68 \ e \ p = 1, ..., 128;$$
 (2)

Where, VBP_s^{UF} is the sectoral Gross Output of each UF. Subsequently, the iterative RAS procedure is used to ensure consistency of the Regional Supply Tables with the Regional Accounts and the National Supply Table. This procedure alters the initial assumption that regional production structures are the same as the national structure.



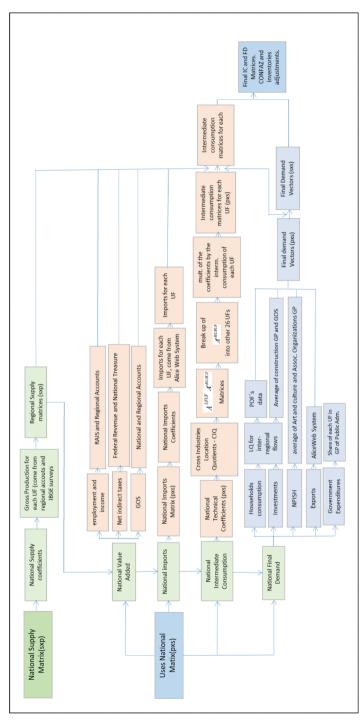


Figure 1 - Process of building the interregional system for 27 Brazilian States using the SUIT method

Source: Research data

The next step is the construction of the Use Tables (product x sector) for the 27 UFs. The Use Tables record the quantity of products that each sector uses as input to carry out its production.

The Regional Gross Output, by product and by sector, were already estimated in the Regional Supply Tables. The share of each state in national imports is obtained from the Foreign Trade Information System - AliceWeb. If there is no information about some product or service, the following Equation is used:

$$m_p^{UF} = m_p^N \frac{z_p^{UF} + d_p^{UF}}{z_p^N + d_p^N} \tag{3}$$

Where m_p^{UF} is the import of the product p by the UF; m_p^N is the national import of the product p; z_p^{UF} is the intermediate demand of the product p in the UF; z_p^N is the national intermediate demand of the product p; d_p^{UF} is the final demand of the product p in the UF and d_p^N is the national final demand of the product p. Essentially, the level of a UF's participation in the national import of a product will align with its contribution to the Gross Production of that UF in comparison to the overall national Gross Production of the specified product.

By AliceWeb system we know which products are being imported by each UF, however, we can't know which sectors of the respective UFs are importing, nor whether this import is for Intermediate Consumption or Final Demand. To solve this problem, it is initially assumed that the use of imports by all UFs follows the national structure, both for Intermediate Consumption and for Final Demand. For this, the National Imports Coefficients $Cimp_{ixi}^N$ are constructed using the National Imports Table:

$$Cimp_{p,s}^{N} = Mimp_{p,s}^{N} * (inv(diag\left(e'_{1,p} * Mimp_{p,s}^{N})\right))$$

$$\tag{4}$$

Where $Mimp_{p,s}^N$ is the National Imports Table and $e'_{1,p}$ is a row vector of ones. The national imports coefficients are then multiplied by the imports by sector of each UF imp_{sx1}^{UF} :

$$Mimp_{p,s}^{UF} = Cimp_{p,s}^{N} * diag(imp_{s,1}^{UF})$$
(5)



In this way, 27 Regional Imports Tables are obtained. Subsequently, the Regional Imports Tables are adjusted by the RAS procedure, ensuring that: (i) the national share of each UF in imports of each product is maintained; and (ii) the sum of the imports of each sector in each UF is the same as the national imports of each sector. Next, the sectoral imports vector by UF is obtained by multiplying:

$$Vimp_{1,s}^{UF} = e'_{1,p} * Mimp_{p,s}^{UF}$$

$$\tag{6}$$

Where $Vimp_{1,S}^{UF}$ is the sectoral import vector for each UF and $e'_{1,p}$ is a row vector of ones. The Value Added elements (by UF and by sector) named: (i) Income; (ii) Gross Operating Surplus (GOS) are estimated based on the Regional Accounts and official surveys such as the National Household Sample Survey – PNAD, and the Annual Social Information Report - RAIS.

Net Indirect Taxes (IIL) are obtained for each UF based on data from: Internal Revenue Service, National Treasury, Social Security and Caixa Econômica Federal. Subsequently, to desegregate these data into 68 sectors, we used the same proportions between Gross Output and Net Indirect Taxes in the national Use Table.

Thus, according to the described procedures we have: (i) Regional Gross Output; (ii) Value Added elements; (iii) imports from the rest of the world; and (iv) Net Indirect Taxes for each UF. Therefore, the following conditions must be met: (i) The national Gross Output must be equal to the sum of the Regional Gross Outputs; (ii) Imports + IIL + VA = GDP from the income approach; (iii) VBP - Imports - IIL - VA = Σ CI, i.e., the sum of Intermediate Consumption of all sectors.

The Final Demand is composed by five elements, namely: (i) Exports; (ii) Household Consumption; (iii) Consumption of Non-Profit Institutions Serving Households - NPISH; (iv) Investments; and (v) Government Expenditures.

Exports by products are obtained from AliceWeb. For products and services whose data are not available on AliceWeb, the following equation is used:

$$e_p^{UF} = e_p^N \frac{x_p^{UF}}{x_p^N} \tag{7}$$

Where e_p^{UF} are regional exports of the product p; e_p^N the national exports of the product p; x_p^{UF} and x_p^N are respectively gross regional and national output. Household Consumption, by UF and by product, is obtained using Family Budget Survey - POF.

The Regional Consumption of NPISH is calculated by a simple average between the participation of each UF in the Gross Output of the following sectors: (i) Art, culture, sports and recreation and other service activities (from Regional Accounts); (ii) and Associative Organizations and Other Personal Services (from the Regional Supply Tables).

The Investment (by product and by UF) follows the procedure: (i) a simple average is made between the share of Regional Gross Output of the Civil Construction sector in the national Gross Output of Civil Construction sector, and the share of Regional Gross Operating Surplus in the national Gross Operating Surplus, in order to obtain the total Investment for each UF from national Investment; (ii) then the total Investment per UF is distributed among the UFs sectors using the structure of the regional Gross Operating Surplus vector; (iii) it is necessary to disaggregate the Investment of each sector in each UF into 128 products, to do so, we used shares from the matrix of investment absorption, according to Miguez et al. (2017).

$$CMinv_{p,s}^{N} = Minv_{p,s}^{N} * inv(diag(e'_{1,p} * Minv_{p,s}^{N}))$$
(8)

$$FBCF_{p,s}^{UF} = CMinv_{p,s}^{N} * diag(fbck_{1,s}^{UF})$$
(9)

$$fbcf_{1,p}^{UF} = FBCF_{p,s}^{UF} * e_{s,1}$$

$$\tag{10}$$

Where, $Minv_{p,s}^N$ is the investment absorption matrix; $e_{s,1}$ and $e'_{1,p}$ are vectors of ones, $fbck_{1,s}^{UF}$ is the Investment vector by sector for each UF, and $fbcf_{1,p}^{UF}$ is the Investment vector by product that will be inserted in the Regional Use Table of each UF. A RAS procedure is then used to ensure consistency with national tables.

Government Expenditures (by sector, by UF) are estimated based on the participation of each UF in the GDP of the public administration, published by IBGE in the Regional Accounts.



The next element of the Regional Use Tables to be estimated is Intermediate Consumption - CI (product x sector). First, intraregional flows are calculated using Cross Industries Locational Quotient-CIQ.

$$CIQ_{ps}^{UF,UF} = \left(\frac{x_p^{UF}/x_p^N}{x_s^{UF}/x_s^N}\right) \tag{11}$$

Where x_p^{UF} and x_p^{N} are the regional and national Gross Output by product; x_s^{UF} and x_s^{N} are the regional and national Gross Output by sector.

The adjustment of the national coefficient to regional coefficient is not the same for all products, because it is necessary to consider the potential trade of each product, according to the Equation:

$$a_{ps}^{UF,UF} = \begin{cases} (CIQ_{p,s}^{UF}).(a_{ps}^{N})...if...(CIQ_{p,s}^{UF} < X) \\ a_{p,s}^{N}.X.....if...(CIQ_{p,s}^{UF} \ge X) \end{cases}$$
(12)

For the sectors related to (i) Agriculture, (ii) Mineral Extraction and (iii) Manufacturing, we use X=0.95. For the sectors of (i) Public Administration, (ii) Associative Organizations and Personal Services; (iii) Domestic Services, it is used X=1. For the other sectors, X=0.9. These values of X are determined empirically.

After the estimation of the intraregional coefficients for the 27 UFs ($A^{UF,UF}$), the interregional flows are calculated. Initially, we estimate the matrices of technical coefficients corresponding to the flows of each UF with the rest of the UFs (RUFs), according to the Equation:

$$A^N - A^{UF,UF} = A^{RUF,UF} \tag{13}$$

Where A^N is the national matrix of technical coefficients; $A^{UF,UF}$ is the matrix of intraregional technical coefficients for each UF; and $A^{RUF,UF}$ is the interregional coefficient matrix, which shows the trade coefficients of the remaining UFs with each UF.

The next step will be to disaggregate each $A^{RUF,UF}$ in other 26 matrices, one for each origin UF, using the Equation:

$$Partic_{p}^{UFO \to UFD} = \frac{X_{p}^{UFO}}{X_{p}^{N} - X_{p}^{UFD}}$$

$$\tag{14}$$

Where $Partic_p^{UFO \to UFD}$ is the share of the origin UF in purchases of the destination UF; X_p^{UFO} is the gross output of product p in origin UF; X_p^{UFD} is the total output of product p in the destination UF; X_p^N is the national gross output of product p.

Once the interregional matrix is obtained for each UF, they are normalized by the column, so that the sum of each column will be equal to one. The flows in monetary values are obtained by multiplying these matrices by the sectoral intermediate consumption of each UF, according to the Equation:

$$CII_{p,s}^{UF} = CICI_{p,s}^{UF} * diag(CIT_{1,s}^{UF})$$

$$(15)$$

Where, $CII_{p,s}^{UF}$ is the monetary values of interregional intermediate consumption in each UF; $CICI_{p,s}^{UF}$ is the interregional coefficient of intermediate consumption for each UF; CIT_{1xs}^{UF} is a vector of sectoral intermediate consumption in each UF, where the last one is calculated according to the equation:

$$CIT_{1.S}^{UF} = VBP_{1.S}^{UF} - VA_{1.S}^{UF} - ILL_{1.S}^{UF} - import_{1.S}^{UF}$$
 (16)

Where $VBP_{1,s}^{UF}$ is the sectoral Gross Output in each UF; $VA_{1,s}^{UF}$ is the sectoral Value Added in each UF; $ILL_{1,s}^{UF}$ are the sectoral Net Indirect Taxes; and $import_{1,s}^{UF}$ are the sectoral Imports in each UF, all of them previously estimated.

Regarding the intra and interregional flows of Final Demand, we operate under the assumption that there are only flows for Household Consumption and Investment. The method employed for estimating intraregional and interregional flows of these Final Demand components is the Simple Locational Quotient method, applying the same criteria as for intermediate consumption. This involves normalizing the coefficients, ensuring that the column sum equals one, thereby maintaining consistency with the national total.

Inventories are obtained residually. However, specific adjustments are implemented: (i) if the national matrix indicates zero inventory in a sector, the estimated interregional matrix also mandates zero inventory for all states in that sector. In such cases, any differences are redistributed along the row of Intermediate Consumption and Final Demand; (ii) acceptable



differences, up to 20%, are tolerated between the inventories of the national matrix and each UF matrix. Larger disparities are distributed to other states. Additional adjustments are made based on data from the National Finance Policy Council – CONFAZ to refine the estimated matrix further.

Thus, after the estimation of all Regional Supply Tables (sector x product) and all Regional and Interregional Use Tables (product x sector), the Interregional Matrix of Input-output (sector x sector) can be finally calculated.

Each element of the regional Supply Tables is divided by the sum of its respective column to build the coefficients matrices for each UF:

$$CP_{s,p}^{UF} = MP_{s,p}^{UF} * inv(diag\left(e'_{1,s} * MP_{s,p}^{UF}\right)\right)$$
(17)

Where are the coefficients for UFs, $CP_{s,p}^{UF}$, $MP_{s,p}^{UF}$ are the Supply Table of each UF and $e'_{1,s}$ is a vector of ones. Subsequently:

$$\begin{bmatrix} CP_{s,p}^{UF1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & CP_{s,p}^{UF27} \end{bmatrix} * \begin{bmatrix} U_{p,s}^{UF1,UF1} & \cdots & U_{p,s}^{UF1,UF27} \\ \vdots & \ddots & \vdots \\ U_{p,s}^{UF27,UF1} & \cdots & U_{p,s}^{UF27,UF27} \end{bmatrix} = \begin{bmatrix} Z_{s,s}^{UF1,UF1} & \cdots & Z_{s,s}^{UF1;UF27} \\ \vdots & \ddots & \vdots \\ Z_{s,s}^{UF27,UF1} & \cdots & Z_{s,s}^{UF27;UF27} \end{bmatrix}$$
(18)

The resulting Z matrix is the Intermediate Consumption Matrix (sector x sector). The Final Demand elements can also be multiplied by the coefficient Matrix, to obtain the Final Demand (sector x sector).

2.2. Interregional Input-Output Adjustment System – IIOAS

Figure 2 presents the process of building the interregional system using IIOAS, which will be detailed in this section. The interstate system estimated by IIOAS is built using the national Input-Output system and: (i) Gross Output (by sector and by UF) VBP^R ; (ii) Exports (by UF and by sector) X^R ; (iii) Value Added (by UF and by sector) VA^R ; (iv) Investment by UF $INVT^R$; (v) Household Consumption by UF CFT^R ; and (vi) Government Expenditures by UF GGT^R . These data are the same as used in SUIT.

A key step in the IIOAS construction process is the estimation of interstate trade matrices, which are built calculating by sector: (i) domestic regional demand; (ii) regional demand for imports; and (iii) domestic regional supply. To obtain domestic regional demand, the coefficients for each user are constructed from the national Use Tables (sector x sector)²:

$$CCI_{i,j}^{DOM} = Z_{i,j}^{DOM} * \hat{X}_j^{-1}$$

$$\tag{19}$$

Where CCI_{ixj}^N is the national coefficient of domestic intermediate consumption, Z^{DOM} is the national matrix of domestic intermediate consumption, and X_j is the national Gross Output. The coefficients are also calculated for the final demand elements:

$$INV_{i,1}^{DOM} = \frac{inv_i^{DOM}}{INVT^N}; CCF_{i,1}^{DOM} = \frac{cf_i^{DOM}}{CFT^N}; CGG_{i,1}^{DOM} = \frac{gg_i^{DOM}}{GGT^N}$$
(20)

Where inv_i^{DOM} , cf_i^{DOM} and gg_i^{DOM} are, respectively, each element i of the (i) investment (ii) household consumption and (iii) government expenditures, and $INVT^N$, CFT^N , GGT^N are respectively the sum (including taxes) of these respective vectors.

In this paper, unlike SUIT, the IIOAS is built on the structure (sector x sector)



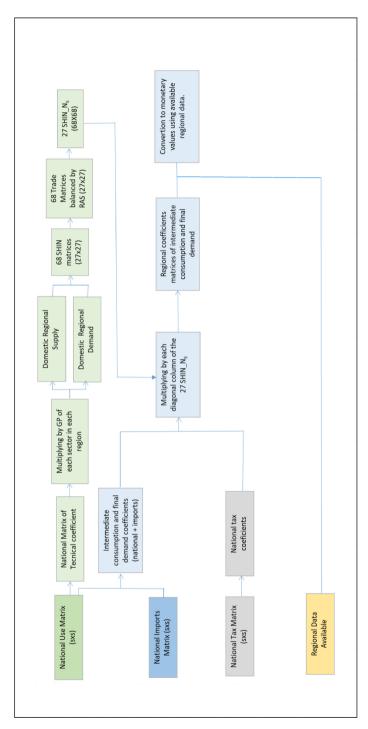


Figure 2 - Process of building the interregional system for 27 Brazilian States using the IIOAS method

Source: Research data

The domestic demand in each UF is obtained by multiplying these coefficients by: (i) Regional Gross Output VBP^R ; (ii) Regional Investment $INVT^R$; (iii) Regional Household Consumption CFT^R ; and (iv) Regional Government Expenditures GGT^R .

$$CI_{i,j}^{R,DOM} = CCI_{i,j}^{DOM} * diag(VBP_{i,1}^{R}) \quad \forall i, j = 1, ..., 68 \text{ e } \forall R = 1,..., 27$$
 (21)

$$INV_{i,1}^{R,DOM} = CINV_{i,1}^{DOM} * INVT_{1,1}^{R} \quad \forall i, j = 1, ..., 68 \text{ e } \forall R = 1,..., 27$$
 (22)

$$CF_{i,1}^{R,DOM} = CCF_{i,1}^{DOM} * CFT_{i,1}^{R} \quad \forall i, j = 1, ..., 68 \text{ e } \forall R = 1,..., 27$$
 (23)

$$GG_{i,1}^{R,DOM} = CGG_{i,1}^{DOM} * GGT_{1,1}^{R} \quad \forall i, j = 1, ..., 68 \text{ e } \forall R = 1,..., 27$$
 (24)

Where $CI_{ixj}^{R,DOM}$ is the regional domestic intermediate consumption, $INV_{i,1}^{R,DOM}$ is the regional domestic Investments, $CF_{i,1}^{R,DOM}$ is the regional domestic household consumption, and $GG_{i,1}^{R,DOM}$ are domestic regional government expenditures. Subsequently, total domestic demand is obtained by summing up:

$$DEMDOM_{i,1}^{R} = \sum_{i=1}^{68} CI_{i,j}^{R,DOM} + INV_{i,1}^{R,DOM} + CF_{i,1}^{R,DOM} + GG_{i,1}^{R,DOM} \quad \forall i = 1, ..., 68 \\ \forall R = 1, ..., 27$$
 (25)

The procedure is similar when regarding the regional demand for imports. The coefficients of demand for imported goods are constructed from the share of each element of the national Imports Table in the totals of each column of the national Uses Table, and later multiplied by the regional totals, following the same procedure of domestic demand.

The regional demand for imported goods is calculated by the sum:

$$DEMDOM_{i,1}^{R} = \sum_{i=1}^{68} CI_{i,j}^{R,IMP} + INV_{i,1}^{R,IMP} + CF_{i,1}^{R,IMP} + GG_{i,1}^{R,IMP} \quad \forall i = 1, ..., 68 \\ \forall R = 1, ..., 27$$
 (26)

This regionalization is consistent with the National Tables, i.e., the sum of $DEMDOM_{i,1}^R$ for all Regions must be equal to the Gross Output of each sector in the national Use Table, without exports. In addition, the sum of $DEMIMP_{i,1}^R$ for all Regions must be equal to the total imports by sector in the national Imports Table.



By putting the vectors of domestic demand $DEMDOM_{i,1}^R$ side by side for all Regions, we have a matrix of dimensions (i,R) where, each row of this matrix represents the domestic demand of one sector i in each of 27 UFs - $DEMDOM_{i,R}$.

Regarding demand for imports, by putting each vector R side by side, we have a matrix (i,R) where each row represents the total imports of sector i by each region R - $DEMIMP_{i,R}$.

The next step is to estimate the domestic regional supply - OFDOM. It is obtained from the difference between Gross Output by sector in each UF VBP^R and exports by sector in each UF X^R .

$$OFDOM_{i,1}^R = VBP_{i,1}^R - X_{i,1}^R \quad \forall i = 1, ..., 68 \text{ e } \forall R = 1, ..., 27$$
 (27)

When putting each regional vector side by side, we have a matrix (ixR) where each row represents the domestic supply of each sector i in each region R.

Subsequently, we build "Share Matrices" (SHIN), which represent the share of each UF in the national trade flows for each sector i. Considering the UF of origin s, and destination d, 68 matrices (one for each sector) of dimension (27x27) are built.

Two equations were used for the construction of these shares. Equation 28 was used to calculate the initial share value of intraregional trade, i.e., the main diagonal of trade matrices. Equation 29 was used to estimate interstate trade flows. Both equations are based on Dixon and Rimmer (2004).

$$SHIN(i,d,d) = Min \left\{ \frac{OFDOM(i,d)}{DEMDOM(i,d)}, 1 \right\} * F$$
(28)

Where SHIN (i,d,d) is the intra-regional share of sector i in national trade. (F) gives the sectoral trade propensity. For sectors 1 to 36, which represent, in general terms, agricultural and industrial production, F=0.5. For sectors 37 to 68, which basically represent services sectors, F=0.95. These values of F are determined empirically. The share of interstate trade flows is defined by the Equation:

$$SHIN(i, s, d) = \left\{ \frac{1}{imped(s, d)} \cdot \frac{OFDOM(i, s)}{\sum_{k=1}^{27} OFDOM(i, k)} \right\}$$

$$* \left\{ \frac{1 - SHIN(i, d, d)}{\sum_{j=1, j \neq d}^{27} \left[\frac{1}{imped(j, d)} \cdot \frac{OFDOM(i, j)}{\sum_{k=1}^{27} OFDOM(i, k)} \right]} \right\}$$

$$(29)$$

Where SHIN (i,s,d) is the share of the trade flow of sector i with origin in region s and destination in region d; the impedance (imped(s,d)) is the average travel time between regions, considering all modals.

After obtaining the SHIN matrices for each sector i (with i=1,...,68) the Trade Matrices were constructed by multiplying each SHIN $(i, s, d)^3$ by its respective value i in the matrix $DEMDOM_{ixR}$.

$$TRADE_{i}^{sd} = SHIN(i, s, d) * diag[DEMDOM_{i,R}(i, 1: R)] \quad \forall i = 1, ..., 68 (30)$$

Where $TRADE_i^{sd}$ are the trade matrices with origin in region s and destination in region d. Then a RAS procedure is used so that the trade matrices converge along the row with the supply, and the column with the demand of sector i for each pair of origin-destination (s, d). Then, it is necessary to include in each $TRADE_i^{sd}$ its respective row i of the matrix $DEMIMP_{i,R}$ including the imports in the regions of origin s.

The Trade matrices reveal how much each Brazilian state sells to each other, and purchases from each other and abroad. However, it is not known whether the purchases in the destination states are for intermediate consumption (in this case, which sector) or final demand.

In order to solve this question, we used a hypothesis originally from multi-regional Chenery-Moses model, proposed by Chenery (1953) and Moses (1955), in which the same trade coefficient is applied for any sector or user in the destination region.

The first step in the regionalization process is to calculate from Trade Matrices a new SHIN_N for each sector *i*:

 $[\]frac{1}{3}$ Where for s = d use SHIN (i, d, d)



$$SHIN_N_{s,d}^{i} = trade_{i}^{sd} \\ * \{inv[diag(\sum_{s=1}^{27} trade_{i}^{sd})]\}$$
 (31)

Where $TRADE_i^{sd}$ is each element of the trade matrices, where s represents the 28 regions of origin (27 national + foreign) and d represents regions of destination (27 national). Subsequently, the elements of National Use Table (sector x sector) are used to construct the national coefficients of intermediate consumption CC^N , investment $CINV^N$, household consumption CCF^N and government expenditures CGG^N . For intermediate consumption:

$$CC_{i,j}^{N} = Z_{i,j}^{DOM+IMP} * (diagCT_{1,j}^{N})^{-1}$$
 (32)

Where $Z_{i,j}^{DOM+IMP}$ is the intermediate consumption matrix, in which each element ij is the sum of the sources: domestic (of the national Uses Table) and imports (of the national Imports Table) and CT_j^N is the total intermediate consumption for each sector j calculated by:

$$CT_{1,j}^{N} = VBP_{1,j}^{N} - VA_{1,j}^{N}$$
(33)

Where $VBP_{1,j}^N$ is the national Gross Output for each sector j and VA_{1xj}^N is the national Value Added for each sector j. Regarding final demand users, each value of each final demand vector is divided by its respective total (including imports and indirect taxes):

$$CINV_{i,1}^{N} = \frac{inv_{i}^{DOM+IMP}}{INVT^{N}}; CCF_{i,1}^{N} = \frac{cf_{i}^{DOM+IMP}}{CFT^{N}}; CGG_{i,1}^{N} = \frac{gg_{i}^{DOM+IMP}}{GGT^{N}}$$
 (34)

Where $inv_i^{DOM+IMP}$ is each value of investment vector, is $cf_i^{DOM+IMP}$ each value of household consumption vector and $gg_i^{DOM+IMP}$ is each value of government expenditures vector (considering domestic + imports sources), and $INVT^N$, CFT^N and GGT^N are the total of the columns of these respective vectors in the national Uses Table.

After that, the regional coefficients are constructed by transforming the 68 SHIN_N (which represent, for each sector, the share of trade flows between each pair origin-destination) into 28 SHIN_S matrices of dimensions 68x27 (which represent, for each origin, including the imports, the share of purchases of each sector in each destination).

Each of the 28 SHIN_S matrices represents an origin. In its rows, the 68 sectors of the economy are disposed, and in its columns the 27 regions of destination. In order to build the regional intermediate consumption coefficients - RCC, each column of each 28 SHIN_S matrix is diagonalized and multiplied by CC_{ixi}^N :

$$RCC_{i,i}^{sd} = diag(SHIN_S(1:i;d)) * CC_{i,i}^{N} \quad \forall d = 1, ..., 27 \text{ e } \forall s = 1,..., 28$$
 (35)

Where s represents the 28 regions of origin and d are the 27 regions of destination. From Equation 35 we can construct 756 matrices (dimension 68x68), which represent the share of each sector in the intermediate consumption of each region of destination.

Regarding the final demand users, the procedure is similar. However, we build for each origin region s, 27 vectors 68x1. These vectors correspond to the share of each 27 destination regions d in the production of 68 sectors of activity.

The demand for investment is:

$$RCINV_{i,1}^{sd} = diag(SHIN_{S(1:i:d)}) * CINV_{i,1}^{N} \quad \forall d = 1, ..., 27 \text{ e } \forall s = 1,..., 28$$
 (36)

For household consumption is:

$$RCCF_{i,1}^{sd} = diag(SHIN_S(1:i;d)) * CCF_{i,1}^{N} \quad \forall d = 1, ..., 27 \text{ e } \forall s = 1,..., 28$$
 (37)

For government expenditures is:

$$RCGG_{i,1}^{sd} = diag(SHIN_S(1:i;d)) * CGG_{i,1}^{N} \forall d = 1, ..., 27 \text{ e} \forall s = 1,..., 28 (38)$$

In order to obtain the regional indirect taxes share, paid by each user, national coefficients are constructed from the national Taxes Table. The tax coefficients are calculated for (i) intermediate consumption, (ii) investment and (ii) household consumption⁴. To regionalize national tax coefficients, SHIN_S matrices are used, in the same way described for intermediate and final consumption.

⁴ For government expenditures, taxes are considered zero.



So as to change regional coefficients into monetary flows between regions, it is necessary to multiply these coefficients by the regional values described at the beginning of this section.

In the interregional input-output system, two Equations are important:

$$VBP_{j}^{R} = \sum_{i=1}^{68} RC_{i,j}^{sd} + \sum_{i=1}^{68} RTC_{i,j}^{sd} + RVA_{j}^{sd}$$
(39)

$$DT_i^R = \sum_{j=1}^{68} RC_{i,j}^{sd} + RINV_i^{sd} + RFC_i^{sd} + XR_i^{sd} + RGG_i^{sd}$$
 (40)

Where VBP_j^R is the Regional Gross Output for sector j; $RC_{i,j}^{sd}$ is the regional Intermediate Consumption; $RTC_{i,j}^{sd}$ are the Indirect Taxes on intermediate consumption, RVA_j^{sd} is the regional Value Added for sector j; DT_i^R is the Total Regional Demand for sector i; $RINV_i^{sd}$ are regional investments; RFC_i^{sd} is regional household consumption; XR_i^{sd} are regional exports; and RGG_i^{sd} are regional government expenditures.

The inventories are obtained by residue:

$$VE_i^R = VBP^R' - DT^R \tag{41}$$

3. Comparison methodology

The comparison of interregional input-output systems can be approached by considering the concepts of partitive and holistic accuracy, as defined by Jensen (1980). Partitive accuracy involves a meticulous cell-by-cell analysis, while holistic accuracy emphasizes the overall "mathematical framework" of economic relations. A higher degree of proximity between the estimated systems signifies greater accuracy. The aim is to discern the methods behavior in evaluating regional economic peculiarities, particularly concerning the productive structure.

Beyond partitive and holistic accuracies, an insightful analysis involves examining the mathematical similarity between estimated interregional input-output systems. According to Lipschutz (1994), a matrix B is similar to a matrix A if there exists a non-singular matrix P such that $B = P^{-1}AP$. Two matrices A and B are considered similar if they share: (i) the same determinant; (ii) the same eigenvalues; (iii) the same trace; and (iv) if A is invertible, B will be so, and if A is singular, B will be as well.

Unlike approaches by Flegg et al. (2016) or Tobben and Kronenberg (2015), this paper does not rely on a matrix derived from a census as a basis for comparison—there isn't a "real interregional system" serving as a reference for identifying the most accurate method. Instead, the objective is to assess whether the systems estimated by SUIT and IIOAS diverge from each other, potentially influencing the results of the input-output analysis.

To evaluate partitive accuracy between estimated systems, two measures will be employed: the Standardized Total Percentage Error (STPE) and the Weighted Absolute Differential (WAD). According to Lahr (2001), these measures are recurrent in the input-output literature.

The STPE was initially applied by Leontief (1986) and later by Jalili (2000) and Lahr (2001) and is defined as:

$$STPE = 100 \frac{\sum_{j} \sum_{i} |a_{ij} - a_{ij}^{*}|}{\sum_{i} \sum_{i} a_{ij}}$$
(42)

Where a_{ij} is the i-th element of the j-th column in the technical coefficients matrix A with dimension (m x n), used as a reference, and a_{ij}^* is the same element of the estimated technical coefficient matrix A^* .

According to Wiebe and Lenzen (2016) STPE is non-symmetric, i.e., the percentage error may be different depending on the matrix used as reference. Therefore, in addition to the traditional STPE, it will be calculated:

$$STPE = 100 \frac{\sum_{j} \sum_{i} |a_{ij} - a_{ij}^{*}|}{\sum_{i} \sum_{i} [(a_{ij} + a_{ij}^{*})/2]}$$
(43)



Thus, the STPE results will be independent of the estimation method used as reference. The WAD, used in Lahr (2001), was developed to correct some problems of other measures, such as non-sensitivity to higher values and the existence of zeros in the matrices (which makes some measures indefinite). It is estimated by:

$$WAD = \frac{\sum_{j} \sum_{i} (a_{ij} + a_{ij}^{*}) * |a_{ij} - a_{ij}^{*}|}{\sum_{i} \sum_{j} (a_{ij} + a_{ij}^{*})}$$
(44)

The term $(a_{ij} + a_{ij}^*)$ weights the absolute difference, giving prominence to errors in larger coefficients. The use of both STPE and WAD proves advantageous as they complement each other. STPE provides differences in percentage, offering an advantage in this regard, but lacks sensitivity to errors in high coefficients. On the other hand, WAD is highly sensitive to discrepancies in high coefficients, yet being an absolute measure, it does not present results in percentage terms.

Holistic accuracy will be gauged through an analysis encompassing: (i) Output multipliers; (ii) a Value Added decomposition of each UF among the final demand of each UF; and (iii) structural decomposition.

Output multipliers elucidate the direct and indirect impacts on output resulting from changes of one monetary unit in the final demand of each sector in each UF. The total multiplier effect is derived by summing the columns of the Leontief inverse matrix⁵. Net output multipliers discount the effect of the injection of one monetary unit and are obtained by subtracting the Leontief inverse from an identity matrix of the same size.

In complementing the analysis of output multipliers, the regional Value Added decomposition concerning the origin of final demand provides additional insights. According to Guilhoto, Siroen, and Yucer (2013), Value Added decomposition may serve as a superior measure compared to Gross Output for understanding the impacts of trade on regional employment and growth.



For the estimation of Leontief inverse matrix, see Miller and Blair (2022)

Considering domestic (v) and external (e) elements of final demand (Y), VA decomposition can be obtained by pre-multiplication of the Leontief inverse matrix by the diagonalized Value Added coefficient - CVA (that is: VA divided by Gross Output).

$$VA^{1} = BCVA^{11}(v^{11} + \dots + v^{UF1} + e^{1}) + \dots + BCVA^{1UF}(v^{1UF} + \dots + v^{UFUF} + e^{UF})$$

$$\vdots \qquad \vdots \qquad \vdots \qquad (45)$$

$$VA^{UF} = BCVA^{UF1}(v^{11} + \dots + v^{UF1} + e^{1}) + \dots + BCVA^{UFUF}(v^{1UF} + \dots + v^{UFUF} + e^{UF})$$

Where *BCVA*¹¹ is the element of the Leontief Inverse matrix multiplied by diagonalized CVA. In this way, we can calculate the share of Value Added (in each UF) that is linked to its own final demand, final demand of other UFs and abroad.

Owen et al. (2014) and Owen (2017) used structural decomposition to analyze the variations in results obtained from the use of interregional input-output systems, built for the same regions, applying different methodologies. The authors used global databases such as WIOD, EORA and GTAP.

For the aim of this study, the structural decomposition of the total Gross Output variation (Δx) between the systems estimated by IIOAS (I) and SUIT (T) can be estimated as:

$$\Delta x = 0.5 \left[\left(\Delta L * YS^{I} * YV^{I} \right) + \left(\Delta L * YS^{T} * YV^{T} \right) \right]$$
 (Tecnological effect) (46)
$$+ 0.5 \left[\left(L^{T} * \Delta YS * YV^{I} \right) + \left(L^{I} * \Delta YS * YV^{T} \right) \right]$$
 (Final demand structure effect)
$$+ 0.5 \left[\left(L^{T} * YS^{T} * \Delta YV \right) + \left(L^{I} * YS^{I} * \Delta YV \right) \right]$$
 (Final demand volume effect)

Where:

$$YS^{I} = DF^{I} * inv[diag(YV^{I})]$$
(47)

$$YS^{T} = DF^{T} * inv[diag(YV^{T})]$$
(48)

$$YV^{I} = e^{i*}DF^{I} \tag{49}$$

$$YV^T = e^{1*}DF^T \tag{50}$$



$$\Delta L = L^T - L^I \tag{51}$$

$$\Delta YS = YS^T - YS^I \tag{52}$$

$$\Delta YV = YV^T - YV^I \tag{53}$$

Where L^T is the SUIT Leontief inverse; DF^T is the SUIT final demand; L^I is the IIOAS Leontief inverse; DF^I is the IIOAS final demand and e' is a row vector of ones used to sum final demands.

The Structural Decomposition method discerns variations between the interregional input-output systems estimated by SUIT and IIOAS. These differences can be attributed to three distinct effects: (i) technology, (ii) final demand structure, and (iii) final demand volume.

In this analytical approach, the cumulative sum of the structural decomposition effects will be zero. This is because the regional Gross Output used in SUIT and IIOAS remains consistent. However, the intraregional and interregional trade flows, as well as the final demand vectors estimated by these two methods, differ. This divergence allows for the identification of each effect independently.

It's crucial to note that this paper does not rely on a matrix as a benchmark for comparison. In other words, there isn't an interregional system serving as a reference. The objective is to assess whether the systems estimated by SUIT and IIOAS for the 27 Brazilian states diverge from each other. Such divergence would imply that the choice of the method can indeed influence the results of the input-output analysis.

4. Results

The Leontief inverse⁶ determinants estimated by SUIT and IIOAS were quite different (Det. IIOAS = 1.016×10^{20} and Det. SUIT = 3.866×10^{26}). The traces, on the other hand, were very similar (IIOAS = 1885.53 and SUIT =1901.06). The mean percentage difference between eigenvalues calculated from Leontief inverse, estimated by IIOAS and SUIT was 2%, with a standard deviation of 2%.

Despite the close values, particularly concerning traces and eigenvalues, it is not appropriate to characterize the Leontief inverses estimated by SUIT and IIOAS as mathematically similar. Nevertheless, this lack of mathematical similarity does not necessarily imply dissimilarity in economic terms.

An initial step involves examining potential biases in the technical coefficients matrices estimated by SUIT and IIOAS. Specifically, the focus is on determining if the technical coefficients matrix estimated by one method consistently exhibits higher/lower values than the matrix estimated by the other method.

The technical coefficient matrix estimated by IIOAS showed that 46% of its values were lower than those estimated by SUIT. Notably, no discernible pattern emerged in the behavior of the coefficients, with those estimated by SUIT consistently higher/lower than those estimated by IIOAS.

Both methods align the GDP of the states (UFs) and the national GDP across the perspectives of (i) expenditure, (ii) production, and (iii) income. However, a slight disparity (approximately 1%) exists in the GDP values for both states and the nation between the methods. This variance is primarily attributed to differences in how each method handles imports and taxes.

In terms of partitive accuracy, the STPE was computed using both the technical coefficients of SUIT and IIOAS, as well as their average as a base.

⁶ Given that the technical coefficients matrices lack invertibility due to correlated rows or columns, we assess the similarity between the Leontief inverses.



The STPE calculated using the average of SUIT and IIOAS coefficients yielded a standardized total percentage error of 54.45% for the entire interregional system. Table 1 presents the results for STPE and WAD in each UF.

Concerning the STPE, the smallest intraregional standard percentage difference between the technical coefficients estimated by SUIT and IIOAS was observed in São Paulo state - SP (23.3%), while the largest occurred in Espírito Santo - ES (49.8%). Considering all UFs, the mean difference between the methods was 38%. For interregional technical coefficients, the disparities were more pronounced, with the highest interregional STPE recorded in Rio de Janeiro - RJ (89%) and the lowest in Mato Grosso - MT (64.6%).

On a broad scale, when SUIT coefficients serve as the base, the mean difference for intraregional flows closely mirrors those calculated using IIOAS coefficients. However, concerning interregional flows, the mean difference, when using SUIT coefficients as the base, is larger than when using IIOAS. This suggests that, in general, the interregional flows estimated by SUIT are lower than those estimated by IIOAS.

According to Oosterhaven (2005) and Barros & Guilhoto (2014), the employment of the simple locational quotient (QL) and interindustry coefficient (CIQ) in SUIT tends to result in an overestimation of intraregional flows and an underestimation of interregional flows. This tendency arises primarily because QL and CIQ, whether implicitly or explicitly, minimize interregional cross-hauling.

Table 1 - Partitive accuracy using STPE and WAD for 27 Brazilian states

	STPE						WAD		
UF	Intrarregional				Inter-reg	gional			
O.	Base SUIT %	Base	Média %	Base SUIT %	Base	Média %	Intrarregional	Interregional	
RO	37.2	42.0	39.4	72.3	65.7	68.8	0.021	0.010	
AC	41.7	45.8	43.7	75.1	72.2	73.6	0.024	0.010	
AM	35.0	31.3	33.0	70.9	68.3	69.6	0.023	0.006	
RR	43.3	45.3	44.3	72.3	73.5	72.9	0.018	0.010	
PA	26.3	28.0	27.1	69.8	69.6	69.7	0.016	0.010	
AP	45.2	43.5	44.3	64.5	67.7	66.1	0.021	0.008	
то	40.8	45.7	43.1	65.4	64.1	64.7	0.020	0.008	
MA	45.8	42.4	44.0	91.9	66.3	77.1	0.027	0.007	
PI	42.0	44.4	43.1	70.9	70.2	70.5	0.020	0.009	
CE	33.2	35.7	34.4	88.0	72.0	79.2	0.018	0.008	
RN	40.6	45.7	43.0	74.9	75.5	75.2	0.021	0.006	
PB	44.0	43.8	43.9	85.1	77.0	80.9	0.015	0.011	
PE	36.2	34.1	35.1	105.6	75.1	87.8	0.016	0.012	
AL	41.9	46.3	44.0	78.4	73.9	76.1	0.025	0.006	
SE	40.4	46.7	43.3	77.8	74.7	76.2	0.030	0.008	
BA	26.7	31.6	29.0	70.8	65.4	68.0	0.018	0.007	
MG	30.9	37.0	33.6	75.1	63.5	68.8	0.022	0.010	
ES	54.0	46.2	49.8	116.7	68.9	86.6	0.023	0.011	
RJ	28.5	35.2	31.5	117.7	71.6	89.0	0.018	0.016	
SP	21.7	25.2	23.3	113.2	70.4	86.8	0.015	0.009	
PR	33.9	39.3	36.4	86.2	71.2	78.0	0.017	0.014	
SC	35.3	32.7	33.9	88.5	64.7	74.7	0.018	0.011	
RS	23.7	26.0	24.8	73.5	65.8	69.5	0.014	0.007	
MS	37.2	39.4	38.3	84.5	66.0	74.1	0.018	0.014	
MT	37.5	43.2	40.1	64.5	64.8	64.6	0.022	0.008	
GO	37.1	42.7	39.7	77.0	63.4	69.6	0.018	0.009	
DF	36.2	44.9	40.0	93.1	66.7	77.7	0.020	0.009	
Average	36.9	39.4	38.0	82.3	69.2	74.7	0.020	0.009	
Max	54.0	46.7	49.8	117.7	77.0	89.0	0.030	0.016	
Min	21.7	25.2	23.3	64.5	63.4	64.6	0.014	0.006	
S-D	7.3	6.7	6.8	15.4	4.1	7.0	0.004	0.003	

Source: Research data



Concerning WAD, Sergipe - SE exhibited the most significant absolute weighted difference (0.030), while Rio Grande do Sul - RS showed the lowest (0.014). It's noteworthy that this measure is highly sensitive to differences between larger coefficients.

The STPE and WAD results highlight that the distinctions in interregional trade flows, estimated by SUIT and IIOAS among Brazilian UFs, are more pronounced than intraregional flows. Despite these larger differences, a remarkably high positive correlation persists between interregional flows estimated by SUIT and IIOAS.

Table 2 presents the Pearson correlation coefficients between interstate trade flows for aggregated sectors of the economy, as estimated by SUIT and IIOAS. The correlation proves to be consistently high across all sectors, including the overall elements of intermediate consumption (94.06%).

Table 2 - Correlation between interregional flows estimated by SUIT and IIOAS for aggregated sectors.

Sectors	Correlation (%)
Agriculture, livestock and forestry	71.18
Manufactures	96.21
Public administration	91.91
Construction	90.15
Sevices	95.88
Total	94.06

Source: Research data

In terms of holistic accuracy, the initial aspect under analysis are the output multipliers. Figure 3 shows the average output multipliers, weighted by Value Added, across the 27 Brazilian states. Notably, the states of Espírito Santo - ES and Santa Catarina - SC exhibit the most significant disparities. In both states, the output multipliers estimated by SUIT are lower than those estimated by IIOAS.

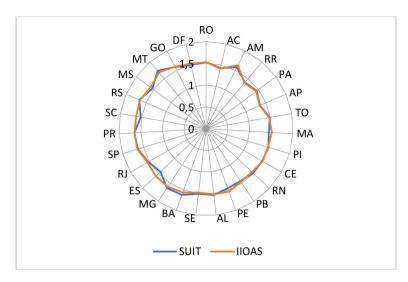


Figure 3 - Average Output Multipliers, weighted by Value Added, estimated from SUIT and IIOAS.

Source: Research data

Regional Value Added and Gross Output remain consistent between SUIT and IIOAS. Therefore, differences in multipliers calculated from SUIT and IIOAS primarily stem from the distinct approaches these methods adopt in handling imports. The correlation is evident: the magnitude of imports directly influences the size of intermediate consumption, thereby impacting the resulting multipliers.

Table 3 provides statistical insights into the differences between output multipliers estimated by SUIT and IIOAS across the 68 sectors within each UF. In the first column, the number of common sectors among the 10 highest output multipliers estimated by each method is displayed. With the exception of Maranhão - MA and Amazonas - AM, for all UFs, this number is either equal to or greater than seven. Additionally, a notable correlation is observed between the sectoral multipliers within each UF.

For nearly all UFs, the mean percentage difference between output multipliers (within the UFs) remained below 2%, though some UFs exhibited a standard deviation higher than the mean.



To complement the information on Table 3, Table 4 presents the percentage differences between interregional and intraregional multipliers, net and total, for each state, estimated from IIOAS and SUIT.

Table 3 - Statistics of the percentage difference between output multipliers estimated from IIOAS and SUIT for Brazilian states.

UF	Common sectors	Correlation Ranking	Correlation coefficients	Max (%)	Min (%)	Average (%)	S-D (%)
RO	9	0.9836	0.9836	3.99	0.00	0.40	0.72
AC	8	0.9912	0.9825	3.68	0.00	0.43	0.62
AM	6	0.9635	0.9752	5.85	0.00	1.12	1.04
RR	9	0.9917	0.9873	3.19	0.00	0.39	0.54
PA	7	0.9833	0.9893	2.95	0.00	0.45	0.47
AP	7	0.9908	0.9892	3.66	0.00	0.41	0.63
то	8	0.9897	0.9893	3.66	0.00	0.50	0.50
MA	5	0.8725	0.8638	12.68	0.00	2.69	2.33
PI	9	0.9889	0.9869	3.98	0.00	0.42	0.59
CE	7	0.9627	0.9725	5.00	0.00	0.71	0.93
RN	8	0.9948	0.9949	1.86	0.00	0.69	0.37
РВ	8	0.9629	0.9664	6.01	0.00	0.90	1.12
PE	7	0.9069	0.9055	10.76	0.00	2.39	1.89
AL	7	0.9922	0.9821	3.06	0.00	0.41	0.53
SE	8	0.9920	0.9933	1.64	0.00	0.55	0.32
ВА	9	0.9943	0.9961	2.29	0.00	1.12	0.49
MG	10	0.9897	0.9921	2.06	0.00	0.57	0.37
ES	7	0.7510	0.7194	17.47	0.00	4.04	3.19
RJ	8	0.9811	0.9856	4.46	0.00	0.67	0.71
SP	8	0.9785	0.9815	3.97	0.00	0.47	0.71
PR	7	0.9840	0.9867	3.71	0.00	0.35	0.59
SC	7	0.8873	0.8889	10.74	0.00	2.44	1.94
RS	9	0.9874	0.9907	2.36	0.00	0.40	0.42
MS	7	0.9402	0.9511	8.40	0.00	1.45	1.45
MT	9	0.9934	0.9921	2.11	0.00	0.92	0.43
GO	8	0.9778	0.9749	5.40	0.00	0.55	0.95
DF	7	0.9675	0.9583	6.36	0.00	1.09	1.12

Source: Research data.

Table 4 - Percentage differences between interregional and intraregional multipliers estimated from SUIT and IIOAS for Brazilian UFs.

UF	TO'	TAL	N	ĒΤ
UF	INTRA (%)	INTER (%)	INTRA (%)	INTER (%)
RO	1.9	4.3	10.0	2.3
AC	1.1	3.0	3.2	8.0
AM	0.3	0.9	7.3	7.0
RR	0.4	1.1	3.6	3.5
PA	0.5	1.4	3.1	0.6
AP	0.9	2.2	10.2	5.9
TO	0.6	1.4	3.7	0.6
MA	7.8	21.9	8.3	2.3
PI	0.2	0.5	1.3	2.8
CE	4.1	14.0	11.3	6.8
RN	0.2	0.6	5.8	1.4
PB	2.6	6.8	2.6	1.0
PE	6.9	24.1	11.9	7.3
AL	1.6	4.8	8.2	2.5
SE	1.1	3.1	8.0	2.5
BA	1.9	6.4	11.7	7.8
MG	3.9	12.8	15.8	11.3
ES	11.4	38.3	16.5	8.2
RJ	8.1	35.9	22.8	26.4
SP	6.3	35.5	16.8	29.4
PR	4.8	15.0	17.7	11.7
SC	6.9	22.9	12.0	6.7
RS	3.1	10.8	11.0	8.4
MS	6.5	17.2	16.8	7.3
MT	0.5	1.2	8.3	2.5
GO	4.9	13.1	17.7	7.8
DF	7.4	21.7	24.0	11.7
Average	3.5	11.9	10.7	6.9
Max	11.4	38.3	24.0	29.4
Min	0.2	0.5	1.3	0.6
S-D	3	12	6	7

Source: Research data

The breakdown of Value Added for each UF attributed to the final demand of that UF is another crucial aspect for holistic accuracy, providing additional insights to complement the analysis of output multipliers. Table 5 illustrates the percentage difference between Regional Value Added associated with the final demand of each UF, as estimated by SUIT and IIOAS.

The most notable differences, whether positive or negative, between the values estimated by SUIT and IIOAS were observed in Bahia - BA, Rio de Janeiro - RJ, and Distrito Federal - DF. In contrast, the lowest differences were noted in São Paulo - SP, Santa Catarina - SC, and Rondônia - RO.



Table 6 shows the percentage variations resulting from differences in (i) technology, (ii) final demand structure, and (iii) final demand volume, between the estimated systems of SUIT and IIOAS. For this analysis, average regional differences in module were employed to prevent negative differences from canceling out positive differences. Additionally, these variations were weighted by the share of regional gross output (for sector *i*) in the total regional gross output, ensuring proportionality in the assessment.

Table 6 - Regional average percentage difference, weighted by the share of regional gross output of the sector i in the total regional gross output.

UF	Tecnological (%)	Final demand structure (%)	Final demand volume (%)	
RO	2.9	3.7	3.4	
AC	3.9	5.4	4.5	
AM	6.2	7.8	4.1	
RR	4.2	6.7	4.9	
PA	4.2	5.1	3.9	
AP	4.6	6.9	4.7	
TO	4.8	6.3	4.4	
MA	5.5	7.4	3.6	
PI	4.0	5.3	4.2	
CE	4.9	6.5	4.1	
RN	3.7	6.7	4.7	
PB	4.4	5.5	4.5	
PE	4.7	5.4	3.7	
AL	6.3	7.1	5.0	
SE	4.2	5.0	4.8	
BA	3.6	3.1	3.1	
MG	2.3	3.5	2.9	
ES	4.8	4.5	2.6	
RJ	3.6	3.6	2.9	
SP	1.9	3.5	2.6	
PR	4.8	5.6	3.3	
SC	4.2	4.8	3.2	
RS	3.3	4.4	2.6	
MS	5.7	7.1	3.2	
MT	4.4	5.6	2.6	
GO	2.7	5.2	3.7	
DF	4.5	9.7	5.6	
Max.	6.32	9.68	5.60	
Min.	1.89	3.15	2.56	
Average	4.23	5.61	3.80	

Source: Research data



According to Table 7, the highest differences between the two methods are in final demand structure. Part of these differences can be attributed to the way in which SUIT and IIOAS deal with inventories.

Table 7- Structure and volume of final demand for SUIT and IIOAS

Final demand	Correlation	SUIT (R\$)	IIOAS (R\$)	Difference (%)
All elements of final demand	0.99	4,387,146	4,352,677	0.7
Investiment	0.99	775,685	774,985	0.1
Household consumption	0.99	2,239,017	2,173,854	3.0
Government expenditures	0.99	817,368	878,087	7.2
Exports	1.00	501,802	501,802	0.0
Inventories	0.31	53,274	23,947	76.0
Inventories (module)	0.30	72,874	125,290	52.9
Inventories (module) service sectors	0.08	248	51,689	198.1

Source: Research data

The inventories estimated through SUIT and IIOAS exhibited a notably low correlation, both in absolute value and module. This correlation diminishes further when focusing solely on the service sectors. Despite this, the inventories in both methods remain sufficiently low, ensuring that they do not compromise the overall totals of the final demand. Notably, the correlation between the sums of the final demand elements reaches a high value of 0.99.

Returning to Table 6, the technological differences stem from variations in the Leontief inverse estimated by SUIT and IIOAS, which, as mentioned earlier, result from the distinctive treatment of imports by each method.

In addition to differences in imports, it is crucial to consider differences in how SUIT and IIOAS estimate their intraregional and interregional trade flows. SUIT determines intraregional trade flows using the CIQ, based on the share of regional Gross Output in national Gross Output for each product. For interregional flows, it also utilizes the share of regional Gross Output of a specific product (in the origin region) in the national production of that product, accounting for the production of the destination region.

In contrast, IIOAS calculates intraregional trade flows using the domestic regional supply and demand ratio. For interregional trade flows, IIOAS employs the share of the origin region's domestic supply in the national domestic supply, multiplied by an impedance coefficient based on travel time between each origin-destination pair.

As a result, SUIT technical coefficients will be lower than IIOAS technical coefficients whenever a region's trade flow is determined by its share in domestic output being lower than that determined by the impedance coefficient. In other words, its share in the national production will be adversely affected by the distance.

5. Final Remarks

This paper aimed to explore the similarities and advantages of interregional input-output systems constructed for the 27 Brazilian UFs using the SUIT and IIOAS methods.

Upon detailing each method, it became evident that both approaches align consistently with the national Input-Output Tables and Regional Accounts. Furthermore, their applicability extends to constructing interregional input-output systems for any country with published Supply and Use Tables and available subnational information for regionalization.

In terms of application, IIOAS exhibits greater ease in dealing with systems among regions of the same level, such as the 27 Brazilian UFs. Conversely, SUIT demonstrates greater flexibility in combining different regional levels. For instance, when applying IIOAS to an inter-regional input-output system containing a municipality, the rest of the state to which it belongs, and the rest of the country, the need arises to construct trade matrices for all municipalities within that state, a step unnecessary when using SUIT.

Evaluating partitive accuracy through STPE and WAD, interregional trade flows exhibited fewer similarities than intraregional trade flows for both SUIT and IIOAS. Despite this, the interregional trade flows estimated by both methods displayed high correlation and low dispersion across all analyzed regions and sectors.



Concerning holistic accuracy between the estimated systems, output multipliers from SUIT and IIOAS demonstrated a high positive correlation. The average difference between sectoral output multipliers within each region remained below 2%, and the ranking of sectoral output multipliers within each region exhibited high correlations, with at least 7 common sectors among the 10 largest estimated multipliers by each system.

The differences in the decomposition of Value Added for each UF linked to the final demand of each UF averaged around 10%. Structural decomposition analysis was conducted to evaluate differences in technology, structure, and volume of final demand. The most substantial average difference was observed in the structure of the final demand (5.6%), primarily attributable to differences in inventories. However, this discrepancy is insufficient to impact the overall total of final demand, as inventories represent a relatively low portion of final demand.

In conclusion, although partitive accuracy suggests seemingly low similarity between the estimated systems, it is insufficient to influence the results of input-output analysis when choosing between an interregional system estimated by SUIT or IIOAS. However, for specific studies focused on particular sectors, especially Agriculture, or regions, particularly in the North of Brazil, analysts should consider the potential variations observed in this study.

References

Barros, G., Guilhoto, J.J.M. 2014. The Regional Economic Structure of Brazil in 1959: An Overview Based on an Interstate Input-Output Matrix. Revista Brasileira de Economia (impresso). 68 (3): 317-335.

Brown, W. M., Bar-Brodeur, A., Tweedle, J. 2020. Firm networks, borders, and regional economic integration. Journal of Regional Science. 60 (2): 374-395.

Dixon, P. B., Rimmer, M. T. 2004. Disaggregation of Results from a Detailed General Equilibrium Model of de US to the State Level. General Working Paper n. 145. Centre of Policy Studies, April.

Domingues, E. P., Haddad, E. A. 2002. Matriz Inter-regional de Insumo-produto Minas Gerais/Resto do Brasil: Estimação e Extensão para Exportações. In: X Seminário sobre a Economia Mineira, Diamantina. Anais do X Seminário sobre a Economia Mineira.

Duan, Y., Dietzenbacher, E., Los, B., Dai, R. 2023. Regional Inequality in China during its rise as a Giant exporter: A value chain analysis. Journal of Regional Science. 63 (1): 148-172.

Flegg, A. T., Mastonardi, L. J., Romero, C.A. 2016. Evaluating the FLQ and AFLQ formulae for estimating regional input coefficients: empirical evidence for the province of Córdoba, Argentina. Economic Systems Research. 18 (1): 21-37.

Gonçalves Jr, C. A., Lopes, R. L., Stamm, C. 2022. The potential impacto f products seized on the western border of Paraná on the Brazilian economy. Regional Science Policy and Practice. 14 (6): 292-307.

Guilhoto, J.J.M., Azzoni, C.R., Ichihara, S.M., Kadota, D.K., Haddad, E.A. 2010. Matriz de Insumo-Produto do Nordeste e Estados: Metodologia e Resultados. Fortaleza: Banco do Nordeste do Brasil. 289 p.

Guilhoto, J.J.M., Gonçalves Jr, C.A., Vicentin, J. C., Imori, D., Ussami, K. A. 2019. Sistema Interestadual de Insumo-Produto do Brasil, Estimação em Condições de Informação Limitada para o ano de 2011: Uma aplicação do Método SUIT. Revista de Economia Aplicada. 23 (1): 83-112.

Guilhoto, J.J.M., Sesso Filho, U.A. 2010. Estimação da Matriz Insumo-Produto Utilizando Dados Preliminares das Contas Nacionais: Aplicação e Análise de Indicadores Econômicos para o Brasil em 2005. Economia & Tecnologia. UFPR/TECPAR. 23, (out./dez.): 53-62.

Guilhoto, J.J.M., Sesso Filho, U.A. 2005a. Estimação da Matriz Insumo-Produto a Partir de Dados Preliminares das Contas Nacionais. Economia Aplicada. 9 (2): 277-299.

Guilhoto, J.J.M.; Sesso Filho, U.A. 2005b. Análise da Estrutura Produtiva da Amazônia Brasileira. Amazônia Ciência e Desenvolvimento, Amazônia, 1, (1): 07-33.

Guilhoto, J.J.M, Siroen, J.M., Yucer, A. 2013. Internal and International Vertical Specialization of Brazilian States – An Input-Output Analysis. 62° Annual Meeting of the French Economic Association. Aix-en-Provence, França.

Haddad, E. A., Faria, W. R., Galvis-Aponte, L. A., Hahn-de-Castro, L. W. 2016. Interregional Input-Output Matriz for Colombia, 2012. Borradores de Economia, 923, Banco de La Republica, Bogotá.

Haddad, E. A., Gonçalves Jr, C. A., Nascimento, T. O. 2017. Matriz Interestadual de Insumo-Produto para o Brasil: Uma Aplicação do IIOAS. Revista Brasileira de Estudos Regionais e Urbanos, 11 (4): 424-446.

Hirschman, A. 1977. Transmissão inter-regional e internacional do crescimento econômico. J. Schwartzman. Economia Regional. Belo Horizonte, Cedeplar.

Ichihara, S. M., Guilhoto, J. J. M. 2008. Geoprocessing and Estimation of Interregional Input-Output Systems an Application to the State of Sao Paulo in Brazil. In: European Regional Science Association, 2008, Liverpool. European Regional Science Association.

Isard, W., Bramhall, D.F. 1988. Methods of inter-regional analysis. Aldershot: Ashgate Publishing, 490p.

Isard, W. 1951. Inter-regional and regional input-output analysis: a model of a space-economy. The Review of Economics and Statistics, Cambridge, 33 (4): 319-328.

Jalili, A. R. 2000. Comparison of Two Methods of Identifying Input-Output Coefficients of Exogenous Estimation. Economic Systems Research, 12 (1): 113-129.

Jensen, R.C. 1980. The Concept of Accuracy in Regional Input-Output Models. International Regional Science Review, 5 (2): 139-154.

Lahr, M. L. 2001. A Strategy for Producing Hybrid Regional Input-Output Tables. In: Lahr, M. L., Dietzenbacher, E. (eds.) Input-Output Analysis: Frontiers and Extensions. New York, Palgrave: 211-242.

Leontief, W., Hollis, B., Chenery, P., Clark, P., Duexenberry J., Ferguson, A., Grosse, R., HlzmanL, M., Isard, W., Kistin H. 1953. Studies in the Structure of the American Economy. White Plains, NY: International Arts and Science Press.

Leontief, W. 1986. Input-Output Economics. Oxford University Press. 2nd Edition, New York.

Miguez, T., Freitas, F. N. P., Squeff, G., Vasconcelos, L., Moraes, T. 2017. Uma Proposta Metodológica para a Estimação da Matriz de Absorção de Investimentos (MAI) para o Período 200-2009. Pesquisa e Planejamento Econômico. Rio de Janeiro, 47 (2): 143-176.

Miller, R.E., Blair, P.D. 2022 Input-output analysis: foundations and extensions. Cambridge University Press, third edition.

Oosterhaven, J. 2005. Spatial interpolation and disaggregation of multipliers. Geographical Analysis.



Owen, A. 2017. Techniques for Evaluating the Differences in Multiregional Input-Output Databases: A Comparative Evaluation of CO2 Consumption-Based Accounts Calculated Using Eora, GTAP and WIOD. Springer International Publishing.

Owen, A., Steen-Olsen, K., Barrett, J., Wiedmann, T., Lenzen, M, A. 2014. Structural Decomposition Approach to Comparing MRIO Databases. Economic Systems Research, 26 (3): 262-283.

Park, J., Gordon, P., Moore II, J. E., Richardson, H.W. 2009. A two-step approach to estimatin state-to-state commodity trade flows. Annals of Regional Science, 43: 1033-1072.

Porsse, A. A., Haddad, E. A., Pontual, E. 2003. Estimando uma Matriz de Insumo-Produto Inter-regional Rio Grande do Sul-Restante do Brasil. In: VI Encontro de Economia Região Sul, 2003, Curitiba. Anpec Sul.

Round, J. I. 1983. Nonsurvey Techniques: A Critical Review of the Theory and the Evidence. International Regional Science Review. 8 (3):189-212.

Tobben, J., Kronenberg, T. 2015. Construction of Multi-Regional Input-Output Tables Using the CHARM Method. Economic Systems Research. 27 (4):487-507.

Wiebe, K. S., Lensen, M. 2016. To RAS or not to RAS? What is the difference in outcomes in multi-regional input-output models. Economic Systems Research, 28 (3): 383-402.

Yamano, N. 2017. Development of Global Inter-Country Inter-Industry System for Varios Policy Perspectives. Phd Dissertation in Urban and Regional Planning. University of Illinois. Urbana-Champaign.

Zhang, Z., Shi, M., Zhao, Z. 2015. The Compilation of China's Interregional Input-Output Model 2002. Economic Systems Research, 27 (2): 238-256.

ACKNOWLEDGMENTS

The authors express their gratitude for the comments provided by anonymous reviewers. Their insightful comments enhanced the quality of the article, contributing to its clarity and overall coherence. All remaining errors are the authors sole responsibility.

CONFLITO DE INTERESSE

Os autores declaram não terem quaisquer conflitos de interesse.

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