An Analysis of the 4th Movement of György Ligeti's *Musica Ricercata* based on Information Theory and Number Partitioning

Adolfo Maia Jr.
Universidade Estadual de Campinas/ Núcleo Interdisciplinar de Comunicação Sonora
adolfo@unicamp.br

Igor Leão Maia
Universidade Federal de Minas Gerais/ Escola de Música
imaia@ufmg.br

Abstract: In this paper we present a mathematically oriented analysis of the 4th Movement of György Ligeti's *Musica Ricercata* (MR). The pitch analysis is based on Theory of Information and rhythm is analyzed through the Theory of Partitions of integer numbers. After a brief historical review of *Musica Ricercata* and its structure we make an analysis of the pitch distribution along the whole MR4 score, as well the Left and Right hand separately, through Theory of Information. We calculate two Information measures, namely, the Shannon's Entropy and the Kullback-Leibler Divergence for the three cases. In the second part, about rhythm, we introduce a simple notation for coding rhythm patterns in terms of partitions of an integer number. We show that, with few exceptions, Ligeti used the partitions of number 6 to get rhythm variations on the Right hand against the ostinato on the Left one. In addition, we show the usefulness of the so called Hasse Diagram as a pre-compositional device to generate rhythm patterns.

Keywords: György Ligeti, Musica Ricercata, Information Theory, Music Analysis, Number Partitioning.

Análise do 4º Movimento de *Musica Ricercata* de György Ligeti baseada em Teoria da Informação e Particionamento Numérico

Resumo: Neste artigo, apresentamos uma análise matematicamente orientada do 4º Movimento da *Musica Ricercata* de György Ligeti (MR4). A análise das alturas é baseada na Teoria da Informação e a do ritmo na Teoria das Partições de números inteiros. Após uma breve revisão histórica de *Musica Ricercata* e sua estrutura, fazemos uma análise da distribuição das alturas ao longo da partitura completa do MR4, bem como da mão esquerda e direita separadamente, através da Teoria da Informação. Calculamos duas medidas de informação, a saber, a Entropia de Shannon e a Divergência de Kullback-Leibler para os três casos. Na segunda parte, sobre ritmo, apresentamos uma notação simples para a codificação de padrões de ritmo em termos de partições de um número inteiro. Mostramos que, com poucas exceções, Ligeti usou as partições do número 6 para obter variações de ritmo na mão direita contra o ostinato na mão esquerda. Além disso, mostramos a utilidade do chamado Diagrama de Hasse como um dispositivo pré-composicional para geração de novos padrões rítmicos.

Palavras-Chave: György Ligeti, Musica Ricercata, Teoria da Informação, Análise Musical, Particionamento de Números.

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I am in prison. One wall is the avant-garde, the other is the past. I want to escape (LIGETI *apud* GRIFFITHS, 2006).

1. Introduction to *Musica Ricercata*

In the Introduction of his comprehensive monograph *György Ligeti: Music of Imagination* Richard Steinitz grasped sensibly the essential nature of Ligeti as a composer

> Ligeti is a dreamer who is a meticulous technician, a creator of mechanism who believes in intuition, an insatiable explorer who is mercilessly self-critical, a hater of ideologies, a profound intellectual who is sensitive, witty, self-deprecating and devoid of pomposity (STEINITZ, 2003, p. xvi).

The Hungarian György Ligeti (1923-2006) is indisputably considered one of the greatest composers of Twentieth Century. He had very personal feeling and opinions about the music of his age as well on the music of the future. This state of affairs, this unconformity, seems to have accompanied Ligeti during all his life. In this work we analyze one of the most famous of his works during the period he yet lived in Hungary, as a music student and later as a young professor at Music Academy of Budapest, namely *Musica Ricercata* for Piano.

In this first period Ligeti's musical style had strong influence from Bartók, Kodály and even Stravinsky in lesser degree, as well from Romanian Folk Music. In the early fifties he managed to produce works in some consonance with the severe restrictions imposed by the authorities of the Communist regime, including being simple and folk-like (KEREFKY, 2008).

Around the early 1950s, a period of great political repression in Hungary under pro Soviet regime, Ligeti changed his mind and started to compose music which no more reflects the musical status. However, he had no opportunity to be officially performed, except partially, in very few cases, due to the strong censorship. Hence, the fate of his *Musica Ricercata* (MR) was a "bottom drawer". In fact, it was premièred only in 1969 in Sweden (STEINITZ, 2003). He clearly mentions the birth of it as a "new music from nothing" and his own style. Firstly, his wish to go beyond Bartok's style and his condition of isolation from external avant-garde influences:

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> About 1950 it became clear to me that developing the post-Bartokian style, in which I had composed before, would not further me. I was twenty-seven years old and lived in Budapest completely isolated from all the ideas, trends, and techniques that had emerged in Western Europe after the war (LIGETI apud KEREFKY, 2008, p. 208).

Secondly, the genesis of *Musica Ricercata* as a problem-solving musical research:

> In 1951 I began to experiment with very simple structures of sonorities and rhythms as if to build up a new kind of music starting from nothing. My approach was frankly Cartesian, in that I regarded all the music I knew and loved as being, for my purpose, irrelevant and even invalid. I set myself such problems as: what can I do with a single note? with its octave? with an interval? with two intervals? What can I do with specific rhythmic relationships which could serve as the basic elements in a formation of rhythms and intervals? Several small pieces resulted, mostly for piano (LIGETI apud STEINITZ, 2003, p. 54).

From these statements it is not difficult to figure out that *Musica Ricercata* was a kind of turning point in Ligeti's creative process. However, it continues to carry many influences from the past, including, material from early works but more importantly it has many musical ideas, as well as materials, which will be reused by Ligeti in his later works.

> *Musica Ricercata* was merely the beginning of a more gradual metamorphosis with lines of continuity that reach back to his original influences and others that lead forward to the innovation that define his later work (LEVY, 2017, p. 13).

Musica Ricercata is a set of eleven short experimental (*ricercata*) pieces for piano, written between October 1951 to March 1953, containing a "minimalist program" of composition. There is an increasing number of allowed pitches for each piece, starting with only two in the first piece and culminating with the last one with twelve pitches in an "idiosyncratic" twelve-tone type approach even before Ligeti had any acquaintance with their protagonists: "I was even totally oblivious of Schoenberg's method of composition with twelve notes, not to mention Webern's procedures" (LIGETI apud STEINITZ, 2003, p. 54).

This way to think about the overall pitch organization shares, roughly, an approach of dodecaphony to envisage a new type of composition system (KEREFKY, 2008). However, the musical language of *Musica Ricercata*, as Ligeti himself affirms, had influences from Bartók, besides from the rich folklore of his country, as shown by these two citations:

> The Pitch scheme is most conspicuous at the beginning of the work, but as more pitches are added in later movements Ligeti begins to rely, once again, on familiar structures, scales, and ideas –especially those from Bartok's musical idiom (LEVY, 2017, p. 13).

> Thus, Ligeti's endeavor to reconcile the twelve-tone system and tonality derives from his idol. No wonder that again we run up against elements of Bartok's style when listening to this pioneering, style-searching composition. At the same time, however, Ligeti openly admits its adherence to his idol by giving the title "Béla Bartók in memorian" to no. IX of *Musica Ricercata* (KEREFKY, 2008, p. 215).

In fact, some authors argue that *Musica Ricercata* has similar characteristics as Bartok's *Mikrokosmos*: both depart from the simplest forms to the increasingly complex ones. Nevertheless, it was worked in a totally different way, being Ligeti's work far more compact and economic in means (TOOP, 1999).

Many musical ideas and materials developed in *Musica Ricercata* were borrowed from early works. Also, some of the materials were reutilized later by Ligeti in several different works (STEINITZ, 2003). The more emblematic one is the arrangement, for the Jeney Quintet, in which he reworked six of them for wind quintet, namely, III, V and VII-X for his *Six Bagatelles for Wind Quintet*. They were approved by the Official Committee in 1953 but were premièred only in 1956, except the last one, Movement X of *Musica Ricercata*, due to censorship at that time. Ironically, nowadays, this is one of the most played pieces of the contemporary wind repertoire.

D. Grantham made an extensive structural analysis of all movements of *Musica Ricercata*, mainly through a descriptive analysis (GRANTHAM, 2014). In addition, Capuzzo presents a comprehensive work on the performance of *Musica Ricercata* and a short interview he got with pianist Pierre-Laurent Aimard, one of the great interpreters of Ligeti's piano music (CAPUZZO, 2015). In fact, Ligeti allows some freedom in the performance of *Musica Ricercata*, and perhaps even something like a machine as Marshal

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suggests apropos of its eleventh movement

> The systematic nature of *Musica Ricercata* made it especially conducive to mechanical performance, and Ligeti indeed authorized an additional adaptation of the Ricercare to barrel organ by Pierre Charial (MARSHALL, 2012, p. 266).

This is also the case of MR4 as Ligeti observes on its score to be played as "à l'orgue de Barbarie" (barrel organ). Some authors go further and interpret those changes in metrics from 3/4 to 2/4 as Ligeti's fine jest of the grinder failing to play the barrel organ or even an imperfection of it (GRANTHAM, 2014). Below we describe its curiously simple and very effective structure.

2. The Overall Structure (Form) of MR4

In this work we are interested in studying some melodic and rhythmic aspects of the fourth movement of *Musica Ricercata* (MR4). This movement presents a waltz like form split in two identical sections. The time signature is 3/4 although, in some few bars, a 2/4 signature is found. So, our analysis can be resumed only for the first part which comprises 60 bars. The form structure of the first part is ABCA. The section A, comprising bars 1-32, presents a two bars simple ostinato on the left-hand and a soft melody on right-hand hovering on it, developed using just four notes from Ligeti's pentatonic. This section is repeated, so we didn't include this repetition in our analysis. Following that, from bars 33-38, there is a small transition section, with chordal variation and slight rhythm variation of the left-hand ostinato, but in higher register. The section B, from bar 38 to 53, shows on the Left hand some sparse variations of the material in section 1 on the same ostinato of section A. section C, from bar 54 to 60, presenting again a chordal structure with the last pitch not yet used G#, has a transitional character being a kind of bridge to return to the same material of the first part and ending the piece. It's worth to mention that the end of this transition section C, at bar 60, has a golden section $r = \frac{1+\sqrt{5}}{2} \cong 0.618$, with the total length of the piece, that is, $60 \times 1.618 \cong 97$. We show below a diagram of the analysis of *Musica Ricercata* IV.

Figure 1 – Diagram showing the formal structure of the paper.

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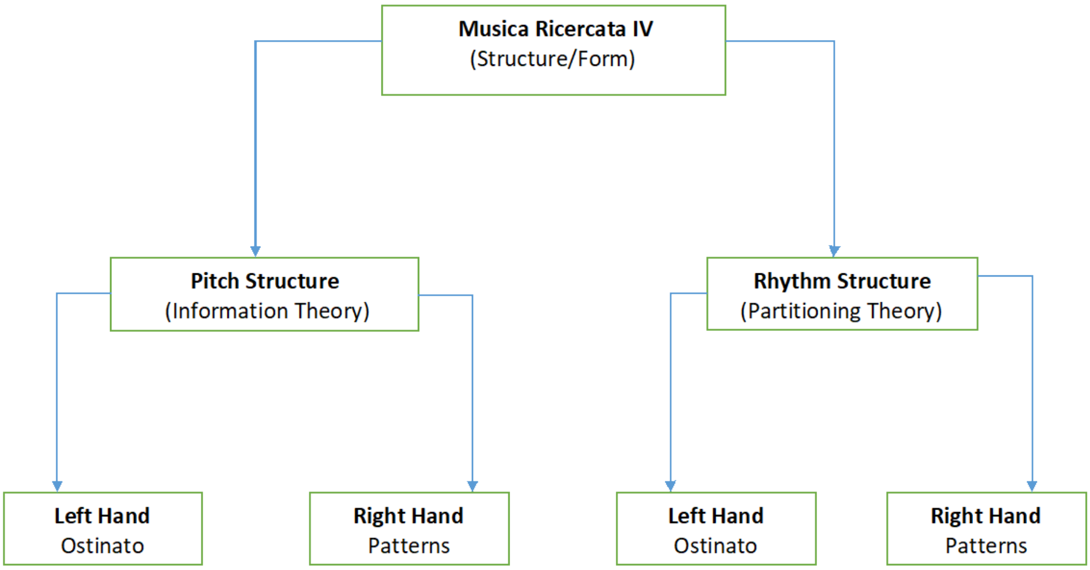
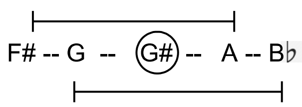


Source: Elaborated by the authors.

3. Pitch Distribution and Structures of MR4

In this section we study briefly the overall form and each one of these sections of MR4. In this short piece, Ligeti uses a chromatic pentatonic, namely, the set ***P*** = {F♯, G, G♯, A, B♭}. Observe in Figure 2 that the note G# is the symmetry center of the pentatonic, despite it is used only in the very short section C which has a transitional character for the return to the beginning of first section.

Figure 2 – G# is the center of symmetry note the pentatonic. The dyads {G, B♭} and {F#, A} are used in the Ostinato having B♭ as a pivot recurrent Bass Note.



Source: Elaborated by the authors.

Nevertheless, its abrupt insertion has a considerable impact on hearing. This is not uncommon Ligeti's expedient in the collection of pieces of *Musica Ricercata*. For example, in MR1 and MR2 a specific note of the set is not used unless in a later and sometimes short section in dramatic, strong dynamics (LEVY, 2017).

Now, except for G#, the other four notes can be considered as a subset of G Minor Harmonic Scale as shown in Figure 3:

Figure 3 – G Minor Harmonic Scale and the Ligeti's subset {G, A, B♭, F#}.

Source: Elaborated by the authors.

Taking this G minor scale into account, observe that {F#, A} "resolves" on the pivot dyad {G, B♭} with a leap of seventh interval. This kind of "resolution", is recurrent on the right-hand "melodic patterns" along the piece since, as mentioned above, it can be considered "written" in G Harmonic Minor. However, MR4 is written in an unconventional mixed key signature, that is F# and B♭, as showed in Figure 4. This kind of mixed Time Signature had already been utilized by Bartok, for example, in Microcosms 70, 99 and 105.

Figure 4 – The left-hand ostinato in the first two bars of *Musica Ricercata* IV showing its Key Signature {F#, B♭}.

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MR4 4 is written in 3/4 "Tempo di valse –, à l'orgue de Barbarie" but once a while a 2/4 metric appears in some sparse bars and, sometimes, even empty. The metronome mark is a dotted half note equal to 96. However, Ligeti observes that interpreter is free for different stylistic idiosyncrasies.

> The metronome value refers to the maximum tempo. The piece may be interpreted freely –as well as being slower– with rubati, ritenuti, accelerandi, just as the organ grinder would play his barrel organ (LIGETI, 1995, p. 13).

3.1 The left-hand ostinato

Ostinati are frequently used in *Musica Ricercata*, such as in MR3, MR4, MR7, MR8. Kerekfy comments its importance in *Musica Ricercata*.

> Another technique concerning the problem of rhythm and meter is the ostinato, which is frequently applied in both *Musica ricercata* and the String Quartet. In connection with the former, Ligeti speaks about the influence of Stravinsky, which is perhaps the most apparent in the different ostinato passages. Ligeti, like Stravinsky, has a predilection for ostinati in which the recurring section is shorter or longer than one bar, or collides some other way with the non-ostinato layers. […] Rhythmic ostinato, i.e., a longer section built upon a single rhythmic pattern, are also frequent. This is the case in both *Musica ricercata* and the String Quartet, in their sections written in giusto syllabique rhythm. Ostinati often go with static harmonization, as pieces IV and VIII of *Musica ricercata* show (KEREFKY, 2008, p. 226).

Some interesting questions can be raised about these ostinato settings. Firstly, using the same pentatonic set, how many different choices, for the two bars ostinato pattern, could Ligeti have made use of? Secondly, keeping intact the rhythm figures along all piece, how these new choices change the overall melodic patterns along the piece? Which impact do these choices have on the psychoacoustic/aesthetic fruition of the piece? These questions can lead to an appreciation of the status of the composer's work relative to its universe of possibilities, that is, relative to similar pieces, as well as his/her compositional choices.

It's important to point out that the first two technical questions are pertinent since an ostinato is, in general, a relatively short pattern and so the number of alternatives is not very large. In this case, it even allows for simulations of alternative pieces, for the sake of comparison. The third question has much more personal bias, thus precluding an objective answer.

In order to answer the first question, we need to do some Combinatorics

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calculations. Observe the Ostinato in Figure 4. In the first bar we have 3 different notes out of 5 possible. The same for the second bar. So, the number of combinations is C(5,3) = 4 and the total number of possibilities is 2 x 4 x 4 = 32, where the factor 2 enter into calculation due the possible exchange of bars 1 and 2.

The answer for the second question depends strongly on the algorithm used to substitute notes. For example, if we consider only non-repetitive substitutions, we must consider only injective (one-to-one) substitution functions defined on the set of Permutations f: ***P*** → ***P***. For example, a function can be viewed as a set of note pairs

{(F#, G), (G, G#), (G#, A), (A, Bb), (Bb, F#)}.

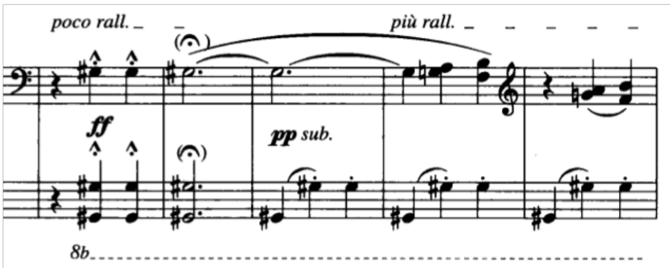
So, it's easy to see that, for 5 notes, the number of possible functions is 5x5 = 25.

This figure is not too big, making it possible for some computer simulation of alternatives, as mentioned above.

For the third question we must stress that there is little chance this or any other function has a comparable musical result as Ligeti's original one. We've made some experiments on this and only a few possibilities merely resemble the beauty of its folk-like melodic flow.

Now, at the second transition section, from bar 54 to 60, after two bars just affirming the new pitch G# in octaves in ***ff***, Ligeti abruptly changes the dynamics to ***pp*** to the same rhythm pattern of the Ostinato but now with its pitch content reduced just to new pitch G#.

Figure 5 – Transition section introducing the fifth note G#.



Source: ©SCHOTT MUSIC, reprinted by permission.

A very minimalist but dramatic bridge section, "placed at the approximate golden section of the movement" (LEVY, 2017, p. 17), after which it returns to the same material. Besides this small section, the piece can be described as a folk-like melody in the right-

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hand, evolving from a simple phrase to sequences of dyads in different octaves, sparse patterns and other horizontal structures, hovering a left-hand ostinato.

After five bars with only the left-hand ostinato, the first part of right-hand melody contains mainly two presentations of six repetitions of four descending notes {Bb, A, G, F#}, since G# is omitted. After that it presents pointillist two note figurations (leaps) of dyads {Bb, G} ending in bar 28 with a dyad {G, F#,}. Ligeti then continues the right-hand with many other variations of these dyads, be them rhythmic, pitch or octave until the beginning of section C on bar 54, with its chordal exploration on the pitch class G#. After that it returns to the beginning of the piece ending with an extended pedal chord {Bb, G} on both hands.

4. Pitch Analysis of MR4: An Information Theory Approach

In this subsection we make some considerations on MR4 from the point of view of Information Theory which was firstly proposed by Claude SHANNON (1948) and since then has been constantly developed and applied to many areas of knowledge (PIERCE, 1980; COVER, 2006).

Applications of Theory of Information and its strongly related subject, Complexity, in Music has a long history. For an account of Information Theory and music, see TEMPERLEY (2007) and KNOPFF (1981). For Complexity in music see TUCKER (2017). Also, in MAIA (2020) there is an application of several Information and Complexity concepts on rhythm patterns.

Information Theory, in practice, is an important tool to analyze patterns of symbolic systems such as human language, computer codes, cryptographic systems, and also music among others (HARRIS, 1991).

The idea here is to consider each note (or even small structures) as a symbol from a suitable alphabet. In this work our alphabet is given by Ligeti's pentatonic ***P*** = {F♯, G, G♯, A, B♭}. We obtain information from a sequence of symbols (in this case, notes of the pentatonic) just counting their appearance in the score and getting their frequencies, which we interpret as probabilities. Now, there exist many different definitions of Information in the literature. By now, we take the first and perhaps simplest of them, the Shannon's Information or Entropy. Shannon's Information, or Entropy, is based on the

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probability that a given symbol appears in a given sequence. Formally, if x is a sequence of symbols from a set, commonly named alphabet, $A = \{a_1, a_2, \ldots, a_N\}$ and p_i is the probability to find the symbol a_i in this sequence. The Shannon Entropy of the sequence x is given by the formula of Equation 1.

$$H(x) = -\sum_{i=1}^{N} p_i log_2 p_i \tag{1}$$

In our case we take as alphabet the pentatonic ***P*** defined above. The allowed sequences are all possible segments from the left or right-hand of the score, or both, or even the entire score. It's worth to observe that probabilities in Shannon Entropy are relative frequencies of the symbols and so the order in which symbols appear in a sequence is inconsequential. This is a severe drawback of the application of Information Theory in Music. A huge number of "similar pieces", many of them musically trivial or perhaps even nonsense, are informationally equivalent to MR4. So, we must consider the results below *cum grano salis*.

Now, it's a theorem from Information Theory that the maximum entropy is achieved when the distribution of probability of symbols is uniform, that is, all symbols of the alphabet have the same probability to appear in a string, or sequence (COVER, 2006). Since we have just 5 symbols, the probability for all of them is $p = \frac{1}{5}$ and so the maximal entropy for any sequence is easily calculated

$$H_0 = -5 \times p\, log_2 p = -5\, \frac{1}{5}\, log_2 \frac{1}{5} = log_2 5 \cong 2.32 \tag{2}$$

This entropy H_0 is our reference. Now, the frequencies that the notes of Ligeti's pentatonic appears in *Musica Ricercata* IV is not random. It's a result of a work by the composer. So, a reduction of Entropy is expected when compared with the random case H_0. The calculation is straightforward. The probability of a symbol (note) is given just by the relative frequency it appears in the score, that is,

$$p(Note) = \frac{N}{N_0} \tag{3}$$

where, N is the number of times the notes (in fact pitch classes) appear in the score, and N_0 is the total number of notes in the score. For example, for the note F# in MR4, $N = 121$, and $N_0 = 653$. In order to count those notes we've made use of a musical

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library provisionally named *maialib*[1]. With this simple definition we get the Table 1, which gives the probabilities for the notes of Ligeti's pentatonic for MR4 (full score), both for left-hand and right-hand.

Table 1 – Probability of Ligeti's pentatonic in MR4, for Left, and Right Hand.

Note	MR4	Left hand	Right hand
F#	$p_1 = \frac{121}{653}$	$p_1 = \frac{67}{389}$	$p_1 = \frac{54}{264}$
G	$p_2 = \frac{170}{653}$	$p_2 = \frac{86}{389}$	$p_2 = \frac{84}{264}$
G#	$p_3 = \frac{25}{653}$	$p_3 = \frac{19}{389}$	$p_3 = \frac{5}{264}$
A	$p_4 = \frac{104}{653}$	$p_4 = \frac{63}{389}$	$p_4 = \frac{41}{264}$
B♭	$p_5 = \frac{233}{653}$	$p_5 = \frac{154}{389}$	$p_5 = \frac{79}{264}$

Source: Elaborated by the authors.

So, the value of Shannon's Entropy of MR4 reads:

$$H_{MR4} = -\sum_{i=1}^{N} p_i log_2 p_i \cong 2.09 \qquad (4)$$

So, the Entropy reduction is just

$$\sigma_{MR4} = \frac{H_{MR4}}{H_0} \cong 0.9 \qquad (5)$$

This can be interpreted as the fact that the distribution of notes in MR4 departs only 10% from a uniform distribution, which is the distribution of random sequences. But, MR4 doesn't seems random at all! What happens with our calculation? In fact, it's very crude! It just counts number of notes. A mathematical approach, say a symbolic system, which is able to grasp and convey all current musical information in a score doesn't exist yet, and probably neither in a near future. The reason is that Information Theory relies on

[1] *maialib* is C++ based library, with a Python wrapper, for Musical Analysis and Composition. It was created by Nycholas Maia and it is under development with its first release expected for December 2021.

a finite alphabet. The smaller, the better. It is a very reductionist approach. The number of symbols to faithfully represent the score could be, in principle, too big as the number of symbols of the score itself, which would imply our analysis as superfluous. Nevertheless, some other information measures can resolve, partially, this problem without any changing in the alphabet.

In fact, a better way to evaluate the above comparison between the two probability distributions is through the *Kullback-Leibler Divergence*, or *Relative Entropy* and it's defined as follows: let $P = \{p_i\}$ and $Q = \{q_i\}$ two discrete probability distributions. The *Relative Entropy* from Q to P is defined as

$$KL(P,Q) = \sum_{i=1}^{N} p_i \log\left(\frac{p_i}{q_i}\right) \tag{6}$$

It can be interpreted as the quantity of information gained if distribution P is used instead Q, or how far the distribution Q is from distribution P. We apply this entropy for the case P is the probabilities got from MR4 as in the above table with $p_i = \frac{1}{5}$ and Q is the Uniform Distribution. Plugin the values in formula we get

$$KL_{MR4}(P,Q) \cong 0.16 \tag{7}$$

Since Q is the Uniform Probability distribution we have, effectively, an entropy reduction around 16% which is more realistic than our former calculation. It's worth to stress again these calculations are only a gross approximation since we didn't take into account many other variables such as rhythm patterns, accentuation, dynamics, among others, which should make the difference between them much more significant and closer to human experience of hear something which is not a "noise" but a sequence of "well-ordered" sounds. The same calculations can be performed to Left and Right Hands. The Table 2 show the results for the three cases.

Table 2 – Information functions for MR4. $H_0 = 2.32$.

Function	**MR4**	**Left Hand**	**Right Hand**
H	$H_{MR4} \cong 2.09$	$H_{lh} \cong 2.08$	$H_{rh} \cong 2.04$
σ	$\sigma_{MR4} = 0.9$	$\sigma_{lh} \cong 0.89$	$\sigma_{rh} \cong 0.88$
KL	$KL_{MR4} \cong 0.16$	$KL_{lh} \cong 0.16$	$KL_{lh} \cong 0.19$

Source: Elaborated by the authors.

The values in Table 2 give us an account of how much the pitches of MR4 departs from a random piece, for piano, of the same size and keeping all other parameters like rhythm, accentuation, etc. It's like we have a MR4 template and we can fill it with different distribution of notes from Ligeti's pentatonic, from the homogeneous random one to Ligeti's own composition. In this respect we have a huge number of possibilities of MR4-like compositions, each one of them with its entropy. The highest entropy is achieved for homogeneous distribution and the lowest one by using just one note (this means probability 1) from the pentatonic all the time. The Ligeti's MR4 is placed between these two extreme cases.

Finally, it's worth to stress that entropy depends direct and strongly on the chosen alphabet. For example, in certain sense in MR4, as well in other movements of *Musica Ricercata*, Ligeti makes use of a smart psychoacoustic device: keep a repetitive pattern (small information ostinato) on the left hand against a higher variable flow on the right one. However, we've calculated the entropy of the left hand and it is not low, 2.08 for a maximum of 2.32. This is because our alphabet are single notes. However, suppose our alphabet is formed with more complex structures, which are closer of our psychoacoustic hearing as, for example, taking as a symbol S of this alphabet is the content of the first two bars of the MR4 Left hand. Since it is repeated many times its probability of S tends to 1 and consequently the information approach to 0. In the psychoacoustic domain we get accustomed with the left-hand repetitive background and pay more attention on the right-hand variations, of course.

5. Partitioning of Integers and Coding for Rhythms

As quoted in the Introduction Ligeti himself pointed out the structural importance of rhythm construction in all movements of *Musica Ricercata*, being the utmost example MR1, in which Ligeti used just two pitch classes.

In this section we present an approach to the rhythm analysis of MR4 based on Partitioning of Integer Numbers and using a suitable coding for its rhythm patterns. More specifically, we analyze mainly right-hand rhythm patterns against the almost ubiquitous rhythm ostinato of the Left hand in Figure 4.

In fact, the right-hand rhythm patterns can be classified, up to some few exceptions, as partitions of the number 6.

Partitions of a natural number n, which we denote by $\Phi(n)$ is the set of all possible ways to write n as a sum of positive integers, where the order not include (Andrews, 1976). A simple example is given by

$$\Phi(4) = \{1+1+1+1,\ 1+1+2,\ 1+3,\ 2+2,\ 4\}.$$

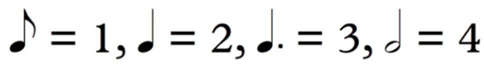
Also, we can use the following notation without the plus sign

$$\Phi(4) = \{1111,\ 112,\ 13,\ 22,\ 4\}.$$

The number of elements of $\Phi(n)$ we denote $P(n)$. So, $P(4) = 5$. Applications of partitions of natural numbers to musical analysis and composition were extensively studied by Gentil-Nunes and collaborators (Gentil-Nunes, 2009, 2010). For $n = 6$ the number of partitions is $P(6) = 11$, which is easy to check.

It turns out the number 6 is the best choice in order to apply Theory of Partitions to the rhythm space Ligeti explores in this piece. This is so, because our analysis takes into account bars and beats in the fixed time signature ¾ and we use the idea of partition of a natural number in a particular way: we code rhythm figures, notes and rests in units of eighth-notes, as follows:

Figure 6 – Units used in the analysis of MR4.



Source: Elaborated by the authors.

The correspondent rests have the same value with negative sign. A whole note has value 8 and a sixteenth note is represented by the fractional value 1/2 and its correspondent rest by -1/2, and so on. However, these fraction values are exceptions in our approach using partitions of integer numbers.

From this coding it is easy to see, in Figure 7, the right-hand bars, from bar 1 to 60, can be coded as small lists of integer numbers using the duration values defined above. Each small list corresponds to a bar, since we are interested in the rhythm variability of the right-hand melody hovering over the left-hand ostinato. The following section, bars

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61-97, is just a repetition of bars 2 to 37 and, therefore, isn't necessary for our analysis.

While the rhythmic ostinato cell of the Left Hand is a balanced pattern |2 2 2|, the right-hand sequence of rhythm patterns s much more varied:

Figure 7 – Rhythm Coding of *Musica Ricercata*'s Right Hand.

$$-6\,|\,-6\,|\,-6\,|\,-4\,|\,-6\,|\,6\,|\,3\,1\,1\,1\,|\,1\,1\,1\,1\,1\,1\,|\,1\,1\,1\,1\,1\,1\,|\,1\,1\,1\,1\,1\,1\,|\,1\,1\,1\,1\,|$$
$$|\,1\,1\,1\,1\,2\,|\,1\,1\,4\,|\,-2\,4\,|\,3\,1\,1\,1\,|\,1\,1\,1\,1\,1\,1\,|\,1\,1\,1\,1\,1\,1\,|\,1\,1\,1\,1\,1\,1\,|\,1\,1\,1\,1\,|$$
$$|\,1\,1\,1\,1\,1\,1\,|\,3\,1\,2\,|\,2\,-2\,2\,|\,4\,2\,|\,-2\,3\,1\,|\,4\,2\,|\,-2\,3\,1\,|$$
$$|\,3\,1\,1\,3/4\,1/4\,|\,4\,2\,|\,-6\,|\,-6\,|\,-4\,|\,-4\,|$$
$$|\,-6\,|\,-4\,|\,1\,1\,4\,|\,6\,|\,6\,|\,-2\,2\,2\,|\,6\,|\,-2\,1\,-1\,-1\,1\,|\,6\,|\,-2\,2\,2\,|\,6\,|$$
$$|\,-2\,1\,-1\,-1\,1\,|\,4\,2\,|\,-4\,|\,-4\,|\,1\,-1\,-1\,1\,2\,|\,1\,1\,1\,-1\,-1\,1\,|$$
$$|\,1\,-1\,2\,2\,|\,6\,|\,-2\,1\,-1\,-1\,1\,|\,4\,2\,|\,-2\,2\,2\,|\,6\,|\,6\,|\,2\,2\,2\,|\,-2\,2\,2\,|\,4\,2\,|\,4\,2\,|$$

Source: Elaborated by the authors.

Observe that, due to the time signature, our choice of the eight-note as time unit, implies $n = 6$ for each bar with few exceptions. If we choose a quarter note, we must take $n = 3$, but this implies too much fractions in the representation and thus partitions don't apply anymore. On the other hand, taking a sixteenth note as time unit implies $n = 12$ and $P(12) = 77$ whose correspondent set of rhythm pattern is much larger than necessary for the analysis of a rhythmically simple piece as MR4 is. So, we think $n = 6$ is the optimal choice for MR4. In our analysis the calculations of partitions do not take into account the difference between note and rest, that is, partitions mean only the division of a bar in time intervals using an integer number of time units. Of course, most works don't fit entirely this requirement and, as mentioned above, some fractional numbers inevitably appear as we can see in Figure 7. As mentioned above, it is possible to circumvent this problem by using a shorter time unit but this implies a bigger number to be partitioned and thus the number of partitions becomes greater, making rhythm analysis far more complex and, in some cases, leading to no meaningful information. So, we consider these cases of fractional numbers as exceptions, as there are only a few of them.

6. A Partitioning Based Rhythm Analysis of MR4

As is the case in the previous movements of *Musica Ricercata*, it is the superposition of rapid changing rhythms by the Right hand against an *ostinato* on the left hand that makes its construction interesting, even using just five pitch classes. Observe that, due to most bars having a time signature of 3/4, the sum of the durations of figures within them, including notes and rests, is 6, according our coding. Ligeti got great rhythm variety by taking different combinations of notes and rests whose durations sum up 6 as shown in Figure 7. In fact, in MR4 Ligeti used 8 partitions out of 11 possible, namely

{1 1 1 1 1 1}, {2 1 1 1 1}, {4 1 1}, {2 2 2}, {3 2 1}, {4 2}, {3 1 1 1}, {6}

Those not appearing in the score are: {2 2 1 1}, {3 3}, {5 1}.

Figure 8 – Bars 41 to 44 with code lists |6 |−222 | 6 | −2 1 −1 −1 1| for the Right hand.

Source: ©SCHOTT MUSIC, reprinted by permission.

Negative values in the lists denote rest durations, and taking this into account the real number of possibilities of rhythm patterns can be greater. For example, if in a partition $n = a_1 + a_2 + \cdots + a_k$ we allow each element to be note (positive value) or rest (negative value) the number of possible permutations (rhythm patterns) is, a priori, $p = 2^k$. Of, course due to element repetitions in some partitions, the rhythm patterns can be less than p.

On the other hand, in contrast with rhythm patterns, partitions are non-ordered sets of numbers. It's easy to see that each partition above has different number of possible associated rhythm patterns, obtained just making permutations between numbers. Formally, each partition is a class of equivalence of rhythm patterns under permutation operation. For example, if we have a partition of a number $n = a_1 + a_2 + \cdots + a_k$ we can make $k!$ permutations of correspondent rhythms associate to it. Other combinatorial operations can be done on the partition and new rhythms can be generated. Of course, Ligeti made use of just a small set of possible rhythm patterns which are representatives of classes of equivalence. Interesting works can be constructed using even just one class of partitions as, for example, Steve Reich's *Clapping Music*: he uses just (restricted) cyclical permutations of the rhythm vector. In binary code it reads [1 1 1 0 1 1 0 1 0 1 1 0] where 1 means a note and 0 a rest. It can be rewritten as [3 2 1 2], just counting the consecutive notes (there is no consecutive rests), adding up to 8. In Clapping Music Reich uses just 12 out of $P(8) = 22$ possible partitions. In terms of composition Reich uses an explicit algorithm (left shifts of the rhythm pattern) and Ligeti, as much as we know, none.

Now, in rhythm analysis an important question is about the distribution and hierarchy of the rhythm patterns along a piece. Our approach in this work, in order to meet this requirement, consists to introduce a kind of taxonomy which is naturally attained by defining an order in the set of rhythm patterns here represented by their correspondent partitions. So, the order (taxonomy) is defined for partitions of an integer number. This can be gotten through the so called *Hasse Diagram* of partitions of an integer number (ZHAO, 2008). The following definition provides a partial order on partitions.

6.1 Partial Order on Partitions

Suppose that $x = (x_1, x_2, \dots, x_r)$ and $y = (y_1, y_2, \dots, y_s)$ are partitions of n. Then x dominates y, written $x \trianglerighteq y$ if

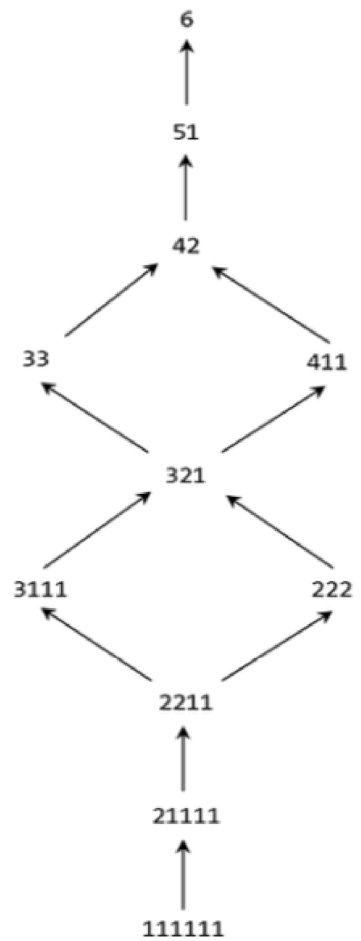
$$x_1 + x_2 \dots + x_i \geq y_1 + y_2 \dots + y_i$$

for all $i \geq 1$. If $i > r$ (respectively, $i > s$), then we take x_i (respectively, y_i) to be zero. This partial order is also named *Dominance Order*.

As an example, by the above definition, we have $\{3\ 3\} \trianglerighteq \{3\ 2\ 1\}$. Nevertheless, it's not possible to compare $\{3\ 3\}$ with $\{4\ 1\ 1\}$ since the inequality fails in both cases of comparison. In this case, we say the partitions are independent of each other. The *Hasse*

Diagram shows the dominance and independence of all partitions as shown in Figure 8 for the case of number $n = 6$ which we use for the analysis of MR4. The dominance is represented by an arrow. Observe that the Hasse Diagram for our case of note (or rest) durations is obtained through augmentation from bottom to top. On the other hand, from top to bottom we have a fragmentation of rhythm patterns.

Figure 9 – *Hasse Diagram* for number 6.



Source: GENTIL-NUNES, 2009.

Following the sequence of partitions for MR4 (as shown in in Figure 6) through Hasse Diagram we find they go through three different sections of the diagram: the first one, at the very outset, its rhythm pattern corresponds to the highest element and suddenly

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moving to the last one. The middle section has great variation of rhythm patterns, which corresponds to a quasi-chaotic path through the central part of the diagram. In the third and last section rhythm patterns concentrates partially at bottom and middle parts of the diagram but returning to its highest parts and ending with partition $\{4\ 2\}$. So, the rhythm patterns show a kind of ABCA' form. Finally, observe that there are 13 out of 60 bars with full rest. This makes the left-hand Ostinato be placed in forefront for the listener.

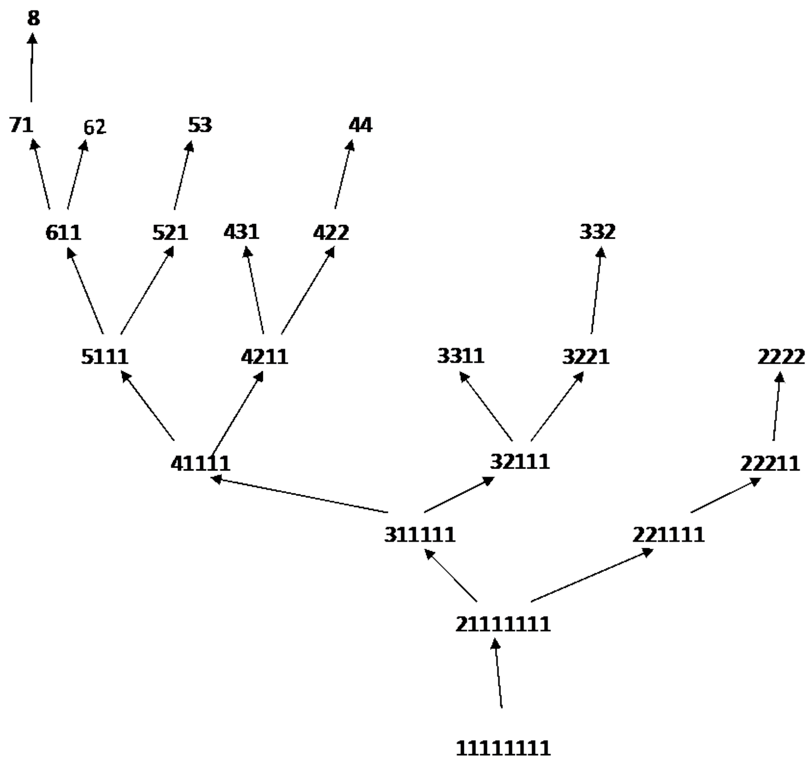
7. Some Examples of Rhythm Pattern Generation

In this section we show a simple example of how the above method can be used as a compositional tool to generate rhythm patterns. Firstly, observe, as mentioned above, that Ligeti didn't use all the partitions of 6 in MR4. In fact, a composition using partitions of a large integer number is likely to explore just a small subset of the partitions. In order to fix such a subset, the composer can use constraints of different kinds, such as symmetry, or even random choices. This economy in terms of rhythm patterns implies, in general, in the use of more complex structures for other musical parameters. In this case the model of rhythm partitions is of little use. Its power is most revealed when the composer intends to use a bunch of varied rhythms constrained by the set of partitions. Below we present some examples of rhythm patterns generated with concatenation and superposition of partitions.

Consider $n = 8$. The number of partitions is $P(8) = 22$ and its Hasse Diagram is shown in Figure 10. Observe that from the bottom to the top, as well from right to left, the nodes growth by augmentation. Since rhythm is an ordered set of durations, we are free to include permutations on any node (rhythm partition) as well as accentuation. In addition, as we did for *Musica Ricercata* IV in Figure 7, we can include rests taking negative values in any partition and its permutations.

Figure 11 shows an example of concatenation of partitions at the extreme limits of Hasse Diagram, namely: [11111111] in the first two bars and [8] in the third one. We've taken a sixteenth note as time unit.

Figure 10 – Hesse Diagram for Partitions of 8.



Source: Elaborated by the authors.

Figure 11 – Using extreme partitions [11111111] and [8] at the edge of Hasse Diagram.

Source: Elaborated by the authors.

Figure 12 shows an example using permutations of partitions from two branches of the Hasse Diagram, where now, the values in a partition count the distance, in time units (in this example, an eighteen note), between chords. The last two Viola and Cello's bars are permutations of the first one.

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Figure 12 – Polyrhythm using different branches of Hasse Diagram.

Source: Elaborated by the authors.

Many other examples can be constructed using the method of time organization above for musical structures in a score. However, observe that for large integer numbers the set of partitions has too many elements. In this case the composer is likely to explore just some branches of Hasse Diagram.

We presented above an extension of Gentil-Nunes' approach to analysis and composition based on Partitions of a positive integer number. Complementing Gentil-Nunes' approach, which uses partitions exploring set of notes, we've applied it to the horizontal dimension of time, that is, rhythm. That approach can be useful in order to search patterns in a score. In fact, the method can be applied to search patterns in time organization of musical structures. However, it is important to stress that our method may not represent the actual rhythm on the score since in our analysis, for example, some artificial adjustments are needed to accommodate partitions, as showed in the analysis of Ligeti's *Musica Ricercata* IV. Nevertheless, this can be thought as a second order rhythm organization. On the other hand, it's possible to extend the above method by just measuring distance, in time units, between structures. These could be single notes, vertical ones such as chords and clusters, horizontal such as rhythm phrases, melodic patterns, or even between any kind of vertical or horizontal blocks.

These simple examples above show that the joint studies of partitions of numbers and associate classes of equivalence under permutations can be a useful tool for analysis and composition of rhythm patterns in other music pieces. The method just asks for a number and returns a bunch of strings. What to do with it is upon the discretion of the composer.

8. Conclusions and Perspectives

In this paper we have used Information Theory for just one independent parameter, namely, pitch distribution. However, even a score, not to mention the interpretation of the piece, has many more parameters, which are articulately superposed in time. So, a comprehensive approach must include a multiple variable Information Theory.

We have also performed a rhythm analysis through Partitioning of Integer Numbers. Theoretically, we can also make an analysis of it based on Information Theory. The basic idea is just to compute the variation of durations of notes along the piece and calculate the Shannon's and Relative Entropy of the left hand to the right hand. It's easy to infer that the right hand has much more rhythm variability than its left counterpart. The problem here is: which is the suitable alphabet of rhythm patterns? This is an interesting topic, since it's related to generating complex rhythm structures as, for example, rhythmic phrases, from basic ones considered as symbols.

Another important aspect of the piece which can be explored is from the point of view of its variability of information in time, which can give us some clues how the musical information is related to psychoacoustic detection of musical structures by the human ear.

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